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Users Guide to the PGAPack Parallel Genetic Algorithm Library

by

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Chapter 0

Quick Start

If you wish to get started by just typing a few lines and running an example, this section is for you. We assume you have `ftp`ed the compressed tar file `pgapack.tar.Z` containing the distribution into `/home/username`. To build a sequential version of PGAPack for a Sun SparcStation in `/usr/local/pga` and run a test example, type

1. `uncompress /home/username/pgapack.tar.Z`
2. `mkdir /usr/local/pga`
3. `cd /usr/local/pga`
4. `tar xvf /home/username/pgapack.tar`
5. `configure -arch sun4`
6. `make install`
7. `/usr/local/pga/examples/c/maxbit`

To build an optimized (no built-in debugging capabilities), parallel version of PGAPack for an IBM SP parallel computer, using an MPI implementation with include files in `/usr/local/mpi/include` and library in `/usr/local/mpi/lib`, and run a test example using four processes, type

1. `uncompress /home/username/pgapack.tar.Z`
2. `mkdir /usr/local/pga`
3. `cd /usr/local/pga`
4. `tar xvf /home/username/pgapack.tar`
5. `configure -arch rs6000 \`
   `-mpiinc /usr/local/mpi/include -mpilib /usr/local/mpi/lib/libmpi.a`
6. `make install`
7. `mpirun -np 4 /usr/local/pga/examples/c/maxbit`

Step 7, the execution step, is completely dependent on the MPI implementation. This example uses the `mpirun` script that is distributed with the `MPICH` implementation [1]. Other MPI implementations may have other ways to specify the number of processes to use.

More details on the installation process and various options are given in Chapter 2. Chapter 3 (example problems) and Sections 5.1 (required functions) and 5.9 (string evaluation and fitness) should be read next.

Part I

Getting Started

Chapter 1

Introduction

PGAPack is a parallel genetic algorithm library that is intended to provide most capabilities desired in a genetic algorithm package, in an integrated, seamless, and portable manner. Key features of PGAPack are as follows:

- Ability to be called from Fortran or C.
- Executable on uniprocessors, multiprocessors, multicomputers, and workstation networks.
- Binary-, integer-, real-, and character-valued native data types.
- Object-oriented data structure neutral design.
- Parameterized population replacement.
- Multiple choices for selection, crossover, and mutation operators.
- Easy integration of hill-climbing heuristics.
- Easy-to-use interface for novice and application users.
- Multiple levels of access for expert users.
- Full extensibility to support custom operators and new data types.
- Extensive debugging facilities.
- Large set of example problems.

Chapter 2

Installation

2.1 Obtaining PGAPack

The complete distribution of PGAPack is available by anonymous ftp from `ftp.mcs.anl.gov` in the file `pub/pgapack/pgapack.tar.Z`. The distribution contains all source code, installation instructions, this users guide, and a collection of examples in C and Fortran. The current release of PGAPack is 1.0. You can check which version of PGAPack you have by running any C language PGAPack program with the command-line option `-pgaversion`.

2.2 Requirements

To compile PGAPack, you *must* have an ANSI C compiler that includes a full implementation of the Standard C library and related header files. If you wish only to build a *sequential* version of PGAPack this is all that is required.

To build a *parallel* version, you *must* have an implementation of the Message Passing Interface (MPI) [5, 6] for the parallel computer or workstation network you are running on. If you do not have a native version of MPI for your computer, several machine-independent implementations are available. Most of the testing and development of PGAPack was done by using the `MPICH` implementation of MPI which is freely available [1].

2.3 Structure of the Distribution Directory

The PGAPack distribution contains the following files and subdirectories:

- **CHANGES:** Changes new to this release of PGAPack.
- **COPYRIGHT:** The usage terms.
- **README:** General instructions, including how to build and install PGAPack.
- **configure.in:** The "source code" for the `configure` script.
- **configure:** A Unix shell script that configures `Makefile.in` for a specific architecture.
- **Makefile.in:** Prototype makefile that is configured into the file `Makefile` for a specific architecture by **configure**.
- **docs:** The users guide and any other supporting files.
- **examples:** A directory containing C and Fortran examples.
- **include:** The PGAPack include directory.

- **lib:** The top-level directory where PGAPack will be installed.
- **man:** The directory containing the PGAPack man pages.
- **source:** The source code for PGAPack.

In the rest of this guide we use "." as the top-level directory, e.g., `./source`, `./examples/c/maxbit.c`.

2.4 Installation Instructions

When installing PGAPack you make two choices: whether to build a sequential (the default) or parallel version (see the flags `-mpiinc` and `-mpilib` below) and whether to build an optimized (the default) or debug version (the `-debug` flag). In broad outline, the installation steps are as follows.

1. Make a directory to install PGAPack in (`mkdir /usr/local/pga`).
2. Change directories to the directory created in the last step (`cd /usr/local/pga`).
3. Obtain the compressed tar file `pgapack.tar.Z` by anonymous ftp from `ftp.mcs.anl.gov` in the directory `pub/pgapack`.
4. Uncompress the tar file (`uncompress pgapack.tar.Z`).
5. Untar the uncompressed PGAPack tar file (`tar xvf pgapack.tar`).
6. Use `configure` to configure the makefiles (`configure -arch ARCH_TYPE`)

   where `ARCH_TYPE` is one of `sun4` for Sun SparcStations workstations, `next` for NeXT workstations, `rs6000` for IBM RS/6000 workstations, `irix` for Silicon Graphics workstations, `hpux` for Hewlett Packard workstations, `alpha` for DEC Alpha workstations, `linux` for machines running Linux, `freebsd` for machines running FreeBSD, `generic` for generic 32-bit machines, `powerchallenge` for the Silicon Graphics Power Challenge Array, `challenge` for the Silicon Graphics Challenge, `t3d` for the Cray T3D, `sp2` for the IBM SP2, `paragon` for the Intel Paragon, or `exemplar` for the Convex Exemplar.

   The full `configure` options are `configure -arch ARCH_TYPE [-cc CC] [-cflags CFLAGS] [-f77 FC] [-fflags FFLAGS] [-debug] [-mpiinc MPI_INCLUDE_DIRECTORY] [-mpilib MPI_LIBRARY] [-help]` where all parameters except `-arch` are optional and do the following:

   - `-cc`: The name of the ANSI C compiler, `cc` by default.
   - `-cflags`: Options passed to the C compiler.
   - `-f77`: The name of the Fortran 77 compiler, `f77` by default. (The Fortran compiler is used only to compile the Fortran examples in the `./examples/fortran` directory.)
   - `-fflags`: Options passed to the Fortran compiler.
   - `-debug`: If specified, enables the debugging features (see Chapter 12) and compiles the source code with the `-g` flag. If this flag is not specified the debugging features are disabled, and the library is compiled with the `-O` flag
   - `-mpiinc`: The *directory* where MPI include files are located.
   - `-mpilib`: The *full path* to the MPI library.

   If `-mpiinc` and `-mpilib` are specified, *a parallel version* of PGAPack will be built. If these flags are not specified, a *sequential version* of PGAPack will be built.
7. Execute the makefile (`make install`).
8. Add PGAPack's man pages to your man page path. (`setenv MANPATH "$MANPATH"":/home/pgapack/man"`)
9. Execute a test problem

- `/usr/local/pga/examples/c/maxbit` in C
- `/usr/local/pga/examples/fortran/maxbit` in Fortran.

If a parallel version of PGAPack was used, the actual commands to execute a parallel program in Step 9 will depend on the particular MPI implementation and parallel computer used. See Appendix D for some examples.

2.5 Installation Examples

These installation examples assume you have `ftp`ed the compressed tar file `pgapack.tar.Z` containing the distribution into `/home/username`.

2.5.1 Sequential Installation

To build a sequential version of PGAPack for a Sun SparcStation in `/usr/local/pga` and run a test example, type:

1. `uncompress /home/username/pgapack.tar.Z`
2. `mkdir /usr/local/pga`
3. `cd /usr/local/pga`
4. `tar xvf /home/username/pgapack.tar`
5. `configure -arch sun4`
6. `make install`
7. `/usr/local/pga/examples/c/maxbit`

2.5.2 Parallel Installation

To build an optimized (no built-in debugging capabilities), parallel version of PGAPack for an IBM SP parallel computer using an MPI implementation with include files in `/usr/local/mpi/include` and library in `/usr/local/mpi/lib`, and run a test example using four processes, type:

1. `uncompress /home/username/pgapack.tar.Z`
2. `mkdir /usr/local/pga`
3. `cd /usr/local/pga`
4. `tar xvf /home/username/pgapack.tar`
5. ```
   configure -arch rs6000 \
   -mpiinc /usr/local/mpi/include -mpilib /usr/local/mpi/lib/libmpi.a
   ```
6. `make install`
7. `mpirun -np 4 /usr/local/pga/examples/c/maxbit`

Step 7, the execution step, is completely dependent on the MPI implementation. This example uses the `mpirun` script that is distributed with the `MPICH` implementation [1]. Other MPI implementations may have other ways to specify the number of processes to use.
   ```

2.6 Mailing Lists, Web Page, and Bug Reporting

To join the PGAPack mailing list to receive announcements of new versions, enhancements, and bug fixes, send electronic mail to `pgapack@mcs.anl.gov`. Bug reports should be sent to `pgapack-bugs@mcs.anl.gov`. The World Wide Web page for PGAPack is `http://www.mcs.anl.gov/pgapack.html` and contains up-to-date news and a list of bug reports.

When reporting a bug, please include as much information and documentation as possible. Helpful information would include PGAPack version number (`-pgaversion`), MPI implementation and version used, configuration options, type of computer system, problem description, and error message output. It is helpful if you put a `PGAPrintContextVariable` call before and after the `PGASetUp` call. Additionally, if possible, build a debug version of PGAPack and send "high-level" output from running your program with the trace facility enabled (Chapter 12).

Chapter 3

Examples

This chapter presents some simple PGAPack programs. The problem chosen is the Maxbit problem. The objective is to maximize the number of 1-bits in a string.

Section 3.1 presents a simple PGAPack program in C whose structure is sufficient to solve many problems. Section 3.2 presents this same program in Fortran. Section 3.3 shows how to change default values in PGAPack. Section 3.4 contains an example that shows how keyboard input may be read in an MPI environment. Finally, Section 3.5 shows how to compile, link, and execute a PGAPack program. These and other examples may be found in the `./examples/c` and `./examples/fortran` directories.

3.1 Maxbit Problem in C

Figure 3.1 shows a minimal program and evaluation function in C for the Maxbit problem. All PGAPack C programs *must* include the header file `pgapack.h`. The `PGACreate` call is always the first function called in a PGAPack program. It initializes the context variable, `ctx`. The parameters to `PGACreate` are the arguments to the program (given by `argc` and `argv`), the data type selected (`PGA_DATATYPE_BINARY`), the string length (`100`), and the direction of optimization (`PGA_MAXIMIZE`). The `PGASetUp` call initializes all parameters and function pointers not explicitly set by the user to default values.

`PGARun` executes the genetic algorithm. Its second argument is the name of a user-defined function (`evaluate`) that will be called to evaluate the strings. `PGADestroy` releases all memory allocated by PGAPack. Note that all PGAPack functions take the context variable as an argument (except `PGACreate`, which creates the context variable).

The `evaluate` function must be written by the user, must return a `double`, and must follow the exact calling sequence shown. `PGAGetStringLength` returns the string length. `PGAGetBinaryAllele` returns the value of the `i`th bit of string `p` in population `pop`.

3.2 Maxbit Problem in Fortran

The Fortran Maxbit problem in Figure 3.2 is similar to the C version in Figure 3.1. The Fortran include file is `pgapackf.h` and should be included in every Fortran function or subroutine that makes PGAPack calls[1]. Since Fortran provides no standard mechanism for specifying command line arguments, these are omitted from the `PGACreate` function call. The context variable, `ctx`, is declared `integer` in Fortran.

The evaluation function `evaluate` must contain exactly the calling sequence shown and must return a `double precision` value. Note that `evaluate` is declared in an `external` statement in the program unit in which it is used as an actual argument. This is a requirement of the Fortran language. In Fortran, the range of allele values is `1:stringlen`, rather than `0:stringlen-1` as in C.

[1] Since not all Fortran compilers support the -I mechanism for specifying the include file search path, you will need to copy or set up a symbolic link to `pgapackf.h` from the directory you are compiling a Fortran program in.

```
#include "pgapack.h"
double evaluate (PGAContext *ctx, int p, int pop);

int main(int argc, char **argv)
{
    PGAContext *ctx;
    ctx = PGACreate (&argc, argv, PGA_DATATYPE_BINARY, 100, PGA_MAXIMIZE);
    PGASetUp        (ctx                                                  );
    PGARun          (ctx, evaluate                                        );
    PGADestroy      (ctx                                                  );
    return;
}

double evaluate (PGAContext *ctx, int p, int pop)
{
    int i, nbits, stringlen;

    stringlen = PGAGetStringLength(ctx);
    nbits     = 0;
    for (i=0; i<stringlen; i++)
        if (PGAGetBinaryAllele(ctx, p, pop, i))
            nbits++;
    return((double) nbits);
}
```

Figure 3.1: PGAPack C Program for the Maxbit Example

```
      include "pgapackf.h"
      external evaluate
      integer ctx
      ctx = PGACreate (PGA_DATATYPE_BINARY, 100, PGA_MAXIMIZE)
      call  PGASetUp  (ctx                                   )
      call  PGARun    (ctx, evaluate                         )
      call  PGADestroy(ctx                                   )
      stop
      end

      double precision function evaluate (ctx, p, pop)
      include "pgapackf.h"
      integer ctx, p, pop, i, bit, nbits, stringlen
      stringlen = PGAGetStringLength(ctx)
      nbits     = 0
      do i=1, stringlen
         bit = PGAGetBinaryAllele(ctx, p, pop, i)
         if (bit .eq. 1) then
            nbits = nbits + 1
         endif
      enddo
      evaluate = dble(nbits)
      return
      end
```

Figure 3.2: PGAPack Fortran Program for the Maxbit Example

```
#include "pgapack.h"
double evaluate (PGAContext *ctx, int p, int pop);

int main(int argc, char **argv)
{
    PGAContext *ctx;
    ctx = PGACreate        (&argc, argv, PGA_DATATYPE_BINARY, 100, PGA_MAXIMIZE);
    PGASetPopSize          (ctx, 500                    );
    PGASetFitnessType      (ctx, PGA_FITNESS_RANKING    );
    PGASetCrossoverType    (ctx, PGA_CROSSOVER_UNIFORM  );
    PGASetUp               (ctx                         );
    PGARun                 (ctx, evaluate               );
    PGADestroy             (ctx                         );
    return;
}
```

Figure 3.3: Specifying Nondefault Values

3.3 Specifying Nondefault Values

PGAPack offers a wide range of choices for parameter values, operators, and algorithmic choices. These will be set to default values in `PGASetUp` if the user does not explicitly set a value for them. A nondefault value may be set by using the `PGASet` family of calls *after* `PGACreate` has been called, but *before* `PGASetUp` has been called.

In Figure 3.3 the `PGASet` calls change the default values for population size, fitness calculation, and crossover type. `PGASetPopSize` changes the population size to 500. `PGASetFitnessType` specifies that the fitness values be determined by using a ranking procedure rather than by direct use of the evaluation function values. `PGASetCrossoverType` specifies that uniform crossover, rather than the default of two-point crossover is to be used. Most `PGASet` calls are discussed in Chapter 5.

3.4 Parallel I/O

The examples in Figures 3.4 (C) and 3.5 (Fortran) read values for the two parameters `len` (string length) and `maxiter` (maximum number of GA iterations) from standard input. These examples will work correctly with either a sequential or parallel version of PGAPack. However, the explicit use of MPI calls for I/O is necessary *only* if a parallel version of PGAPack is used, and parameter values are read from standard input. The purpose is to be sure that each process receives a copy of the input values. See Appendix C for further details.

`MPI_Init(&argc, &argv)` is always the first function called in any MPI program. Each process executes `MPI_Comm_rank(MPI_COMM_WORLD, &myid)` to determine its unique rank in the communicator[2] `MPI_COMM_WORLD`. The logic used in this program is to have process `0` read and write from/to standard input/output and broadcast (using `MPI_Bcast`) the parameters to the other processes. The PGAPack function calls are similar to those in the previous examples. If the user called `MPI_Init`, the user must also call `MPI_Finalize` before exiting.

We elaborate here on the `MPI_Bcast` function because of its practical value in the model of parallel I/O shown. For more detailed discussion of MPI concepts and functions, the user should consult [5, 6].

The C binding for `MPI_Bcast` is

```
int MPI_Bcast(void *buf, int count, MPI_Datatype datatype, int root, MPI_Comm comm)
```

and the Fortran binding

[2] See Appendix C

```
#include "pgapack.h"
double evaluate (PGAContext *ctx, int p, int pop);

int main( int argc, char **argv )
{
     PGAContext *ctx;
     int myid, len, maxiter;

     MPI_Init(&argc, &argv);
     MPI_Comm_rank(MPI_COMM_WORLD, &myid);
     if (myid == 0) {                        /* Process 0 has a dialog */
         printf("String length? ");          /* with the user and      */
         scanf("%d", &len);                  /* broadcasts the user's  */
         printf("Max iterations? ");         /* parameters to all      */
         scanf("%d", &maxiter);              /* other processes        */
     }
     MPI_Bcast(&len,     1, MPI_INT, 0, MPI_COMM_WORLD);
     MPI_Bcast(&maxiter, 1, MPI_INT, 0, MPI_COMM_WORLD);

     ctx = PGACreate(&argc, argv, PGA_DATATYPE_BINARY, len, PGA_MAXIMIZE);
     PGASetMaxGAIterValue(ctx, maxiter);
     PGASetUp(ctx);
     PGARun(ctx, evaluate);
     PGADestroy(ctx);

     MPI_Finalize();
     return(0);
}
```

Figure 3.4: PGAPack Maxbit Example in C with I/O

```
      include 'pgapackf.h'
      include 'mpif.h'

      double precision evaluate
      external         evaluate

      integer ctx, myid, len, maxiter, ierror

      call MPI_Init(ierror)
      call MPI_Comm_rank(MPI_COMM_WORLD, myid, ierror)

c     Process 0 has a dialog with the user and broadcasts the user's
c     parameters to all other processes
      if (myid .eq. 0) then
         print *, 'String length?'
         read  *, len
         print *, 'Max iterations?'
         read  *, maxiter
      endif
      call MPI_Bcast(len,     1, MPI_INT, 0, MPI_COMM_WORLD, ierror)
      call MPI_Bcast(maxiter, 1, MPI_INT, 0, MPI_COMM_WORLD, ierror)

      ctx = PGACreate(PGA_DATATYPE_BINARY, len, PGA_MAXIMIZE)
      call PGASetMaxGAIterValue(ctx, maxiter)
      call PGASetUp(ctx)
      call PGARun(ctx, evaluate)
      call PGADestroy(ctx)

      call MPI_Finalize(ierror)

      stop
      end
```

Figure 3.5: PGAPack Maxbit Example in Fortran with I/O

```
MPI_BCAST(buffer, count, datatype, root, comm, ierror)
<type> buffer(*)
integer count, datatype, root, comm, ierror
```

`MPI_Bcast` will result in every process in communicator `comm` receiving a copy of the contents of `*buf/buffer`. The other parameters are the number of items (`count`), the datatype (`datatype`), which may be one of `MPI_DOUBLE`, `MPI_INT`, `MPI_CHAR`, `MPI_UNSIGNED`, or `MPI_LONG`; the rank of the process with the original copy (`root`); the MPI communicator (`comm`); and, for Fortran, a variable to handle an error return code (`ierror`).

3.5 Compiling, Linking, and Execution

When PGAPack was installed, the makefiles in the `./examples/c` and `./examples/fortran` directories were correctly configured for the machine PGAPack was installed on using the version of MPI specified (if any). To run your own programs, it is best to copy the appropriate makefile (C or Fortran) to your directory and modify it to use your source code files. The makefile will compile your source code files, link in the PGAPack library (and MPI library if a parallel version of PGAPack was built), and build your executable.

How you execute your program will depend on whether a sequential or parallel version of PGAPack was

built, the MPI implementation used and the machine you are running on. If a sequential version of PGAPack was built (i.e., one where the user did not supply a version of MPI), the executable `maxbit` can be executed on a uniprocessor Unix system by typing `maxbit`. If the `MPICH` implementation of MPI was used, it may be executed (using four processes) by `mpirun maxbit -np 4`. Appendix D contains some examples.

Part II

Users Guide

Chapter 4

The Structure of PGAPack

This chapter provides a general overview of the structure of PGAPack.

4.1 Native Data Types

PGAPack is a data-structure-neutral library. By this we mean that a data-hiding capability provides the full functionality of the library to the user, in a transparent manner, irrespective of the data type used. PGAPack supports four native data types: binary-valued, integer-valued, real-valued, and character-valued strings. In addition, PGAPack is designed to be easily extended to support other data types (see Chapter 7).

The binary (or bit) data type (i.e., `|1|0|1|1|`) is the traditional GA coding. The bits may either be interpreted literally or decoded into integer or real values by using either binary coded decimal or binary-reflected Gray codes. In PGAPack the binary data type is implemented by using each distinct bit in a computer word as a gene, making the software very memory-efficient. The integer-valued data type (i.e., `|3|9|2|4|`) is often used in routing and scheduling problems. The real-valued data type (i.e., `|4.2|7.1|-6.3|0.8|`) is useful in numerical optimization applications. The character-valued data type (i.e., `|h|e|l|l|o|w|o|r|l|d|`is useful for symbolic applications.

4.2 Context Variable

In PGAPack the *context variable* is the data structure that provides the data hiding capability. The context variable is a pointer to a C language structure, which is itself a collection of other structures. These (sub)structures contain all the information necessary to run the genetic algorithm, including data type specified, parameter values, which functions to call, operating system parameters, debugging flags, initialization choices, and internal scratch arrays. By hiding the actual data type selected and specific functions that operate on that data type in the context variable, user-level functions in PGAPack can be called independent of the data type.

Almost all fields in the context variable have default values. However, the user can set values in the context variable by using the `PGASet` family of function calls. The values of fields in the context variable may be read with the `PGAGet` family of function calls.

4.3 Levels of Usage Available

PGAPack provides multiple levels of control to support the requirements of different users. At the simplest level, the genetic algorithm "machinery" is encapsulated within the `PGARun` function, and the user need specify only three parameters: the data type, the string length, and the direction of optimization. All other parameters have default values. At the next level, the user calls the data-structure-neutral functions explicitly (e.g., `PGASelect, PGACrossover, PGAMutation`). This mode is useful when the user wishes more explicit control over the steps of the genetic algorithm or wishes to hybridize the genetic algorithm with a

hill-climbing heuristic. At the third level, the user can customize the genetic algorithm by supplying his or her own function(s) to provide a particular operator(s) while still using one of the native data types. Finally, the user can define his or her own datatype, write the data-structure-specific low-level GA functions for the datatype (i.e., crossover, mutation, etc.), and have the data-structure-specific functions executed by the high-level data-structure-neutral PGAPack functions.

4.4 Function Call–Based Library

All the access to, and functionality of, the PGAPack library is provided through function calls.

- The `PGASet` family of functions sets parameter values, allele values, and specifies which GA operators to use. For example, `PGASetPopSize(ctx,500)` sets the GA population size to 500.
- The `PGAGet` family of functions returns the values of fields in the context variable and allele values in the string. For example, `bit = PGAGetBinaryAllele(ctx,p,pop,i)` returns the value of the `i`th bit in string `p` in population `pop` into `bit`.
- The simplest level of usage is provided by the `PGARun` function. This function will run the genetic algorithm by using any nondefault values specified by the user and default values for everything else.
- The next level of usage is provided by the data-structure-neutral functions, which the user can call to have more control over the specific steps of the genetic algorithm. Some of these functions are `PGASelect, PGACrossover, PGAMutate, PGAEvaluate`, and `PGAFitness`.
- The data-structure-specific functions deal directly with native data types. In general, the user never calls these functions directly.
- System calls in PGAPack provide miscellaneous functionality, including debugging, random number generation, output control, and error reporting.

4.5 Header File and Symbolic Constants

The PGAPack header file contains symbolic constants and type definitions for all functions and should be included in any file (or function or subroutine in Fortran) that calls a PGAPack function. For example, `PGA_CROSSOVER_UNIFORM` is a symbolic constant that is used as an argument to the function `PGASetCrossoverType` to specify uniform crossover. In C the header file is `pgapack.h`. In Fortran it is `pgapackf.h`

4.6 Evaluation Function

PGAPack requires that the user supply a function that returns an evaluation of a string that it will map to a fitness value. This function is called whenever a string evaluation is required. The calling sequence and return value of the function must follow the format discussed in Section 5.9.

4.7 Parallelism

PGAPack can be run on both sequential computers (uniprocessors) and parallel computers (multiprocessors, multicomputers, and workstation networks). The parallel programming model used is message passing, in particular the single program, single data (SPMD) model. PGAPack version 1.0 supports sequential and parallel implementations of the global population model (see Chapter 10).

4.8 Implementation

PGAPack is written in ANSI C. A set of interface functions allows most user-level PGAPack functions to be called from Fortran. All message-passing calls follow the Message Passing Interface (MPI) standard [5, 6]. Nonoperative versions of the basic MPI functions used in the examples are supplied if the user does not provide an MPI implementation for their machine. These routines simply return and provide *no* parallel functionality. Their purpose is to allow the PGAPack library to be built in the absence of an MPI implementation.

Most low-level internal functions in PGAPack are data-structure *specific* and use addresses and/or offsets of the population data structures. The user-level routines, however, provide the abstractions of data-structure *neutrality* and an integer indexing scheme for access to population data structures.

Chapter 5

Basic Usage

As the examples in Chapter 3 show, a PGAPack program can be written with just four function calls and a string evaluation function. This basic usage is discussed further in Section 5.1. Sections 5.3–5.12 explain options available in PGAPack. Section 5.13 discusses PGAPack command line arguments.

5.1 Required Functions

Any file (or function or subroutine in Fortran) that uses a PGAPack function must include the PGAPack header file. In C this file is `pgapack.h`. In Fortran this file is `pgapackf.h`. The first PGAPack call made is *always* to `PGACreate`. In C this call looks like

```
PGAContext *ctx;
ctx = PGACreate (&argc, argv, datatype, len, maxormin);
```

`PGACreate` allocates space for the context variable, `ctx` (Section 4.2), and returns its address. `argc` and `argv` are the standard list of arguments to a C program. `datatype` must be one of `PGA_DATATYPE_BINARY`, `PGA_DATATYPE_INTEGER`, `PGA_DATATYPE_REAL`, or `PGA_DATATYPE_CHARACTER` to specify strings consisting of binary-valued, integer-valued, real-valued, or character-valued strings, respectively. `len` is the length of the string (i.e., the number of genes). `maxormin` must be `PGA_MAXIMIZE` or `PGA_MINIMIZE` to indicate whether the user's problem is maximization or minimization, respectively.

In Fortran the call to `PGACreate` is

```
integer ctx
ctx = PGACreate (datatype, len, maxormin)
```

where `datatype`, `len`, and `maxormin` are the same as for C programs. After the `PGACreate` call, the user may *optionally* set nondefault values. These are then followed by a call to `PGASetUp` to initialize to default values all options, parameters, and operators not explicitly specified by the user. For example,

```
ctx = PGACreate(&argc, argv, datatype, len, maxormin);
PGASetPopSize              (ctx, 500);
PGASetFitnessType          (ctx, PGA_FITNESS_RANKING);
PGASetCrossoverType        (ctx, PGA_CROSSOVER_UNIFORM);
PGASetUniformCrossoverProb (ctx, 0.6);
PGASetUp                   (ctx);
```

will change the default values for the population size, the mapping of the user's evaluation to a fitness value, and the crossover type. All `PGASet` calls should be made *after* `PGACreate` has been called, but *before* `PGASetUp` has been called; all such calls are *optional*. Note also that *all* PGAPack functions other than `PGACreate` take the context variable as their first argument.

The `PGARun` function executes the genetic algorithm. Its second argument is the name of a user-supplied evaluation function that returns a `double` (`double precision` in Fortran) value that is the user's evaluation of an individual string. In C the prototype for this function looks like

```
double evaluate (PGAContext *ctx, int p, int pop);
```

and in Fortran

```
double precision function evaluate (ctx, p, pop)
integer ctx, p, pop
```

The user *must* write the evaluation function, and it *must* have the calling sequence shown above and discussed further in Section 5.9. After `PGARun` terminates, `PGADestroy` is called to release all memory allocated by PGAPack. [1]

Except for writing an evaluation function (Section 5.9) the information contained in rest of this chapter is optional—defaults will be set for all other GA parameters. We do note, however, that the defaults used are the result of informal testing and results reported in the GA literature. *They are by no means optimal*, and additional experimentation with other values may well yield better performance on any given problem.

5.2 Population Replacement

Two population replacement schemes are common in the literature. The first, the *generational replacement* genetic algorithm (GRGA), replaces the entire population each generation and is the traditional genetic algorithm [7]. The second, the *steady-state* genetic algorithm (SSGA), typically replaces only a few strings each generation and is a more recent development [9, 10, 11]. PGAPack supports both GRGA and SSGA and variants in between via *parameterized* population replacement. For example, the `PGASet` calls

```
PGASetPopSize           (ctx,200);
PGASetNumReplaceValue   (ctx,10);
PGASetPopReplacementType(ctx, PGA_POPREPL_BEST);
```

specify that each generation a new population is created consisting of ten strings created via recombination, and the 190 most fit strings from the old population. The 190 strings can also be selected randomly, with or without replacement, by setting the second argument of `PGASetPopReplacementType` to `PGA_POPREPL_RANDOM_REP` or `PGA_POPREPL_RANDOM_NOREP`, respectively.

By default, the number of new strings created each generation is 10 percent of the population size (an SSGA population replacement strategy). A GRGA can be implemented by setting `PGASetNumReplaceValue` to the population size (the default population size is 100). Setting `PGASetNumReplaceValue` to one less than the population size will result in an elitist GRGA, where the most fit string is always copied to the new population (since `PGA_POPREPL_BEST` is the default population replacement strategy).

Traditionally, strings created through recombination first undergo crossover and then mutation. Some practitioners [3] have argued that these two operators should be separate. By default, PGAPack applies mutation only to strings that did *not* undergo crossover. This is equivalent to setting `PGASetMutationOrCrossoverFlag (ctx,PGA_TRUE)`. To have strings undergo *both* crossover and mutation, on should use `PGASetMutationAndCrossoverFlag (ctx,PGA_TRUE)`.

By default, PGAPack allows duplicate strings in the population. Some practitioners advocate not allowing duplicate strings in the population in order to maintain diversity. The function call `PGASetNoDuplicatesFlag (ctx,PGA_TRUE)` will not allow duplicate strings in the population: It repeatedly applies the mutation operator (with an increasing mutation rate) to a duplicate string until it no longer matches any string in the new population. If the mutation rate exceeds 1.0, however, the duplicate string *will* be allowed in the population, and a warning message will be issued.

Figure 5.1 shows the generic population replacement scheme in PGAPack. Both populations k and $k+1$ are of fixed size (the value returned by `PGAGetPopSize`). First, `PGAGetPopSize` - `PGAGetNumReplaceValue` strings are copied over directly from generation k. The way the strings are chosen, the most fit, or randomly with or without replacement, depends on the value set by `PGASetPopReplacementType`. The remaining `PGAGetNumReplaceValue` strings are created by crossover and mutation.

[1] `PGADestroy` will also call `MPI_finalize`, if MPI was started by `PGACreate`.

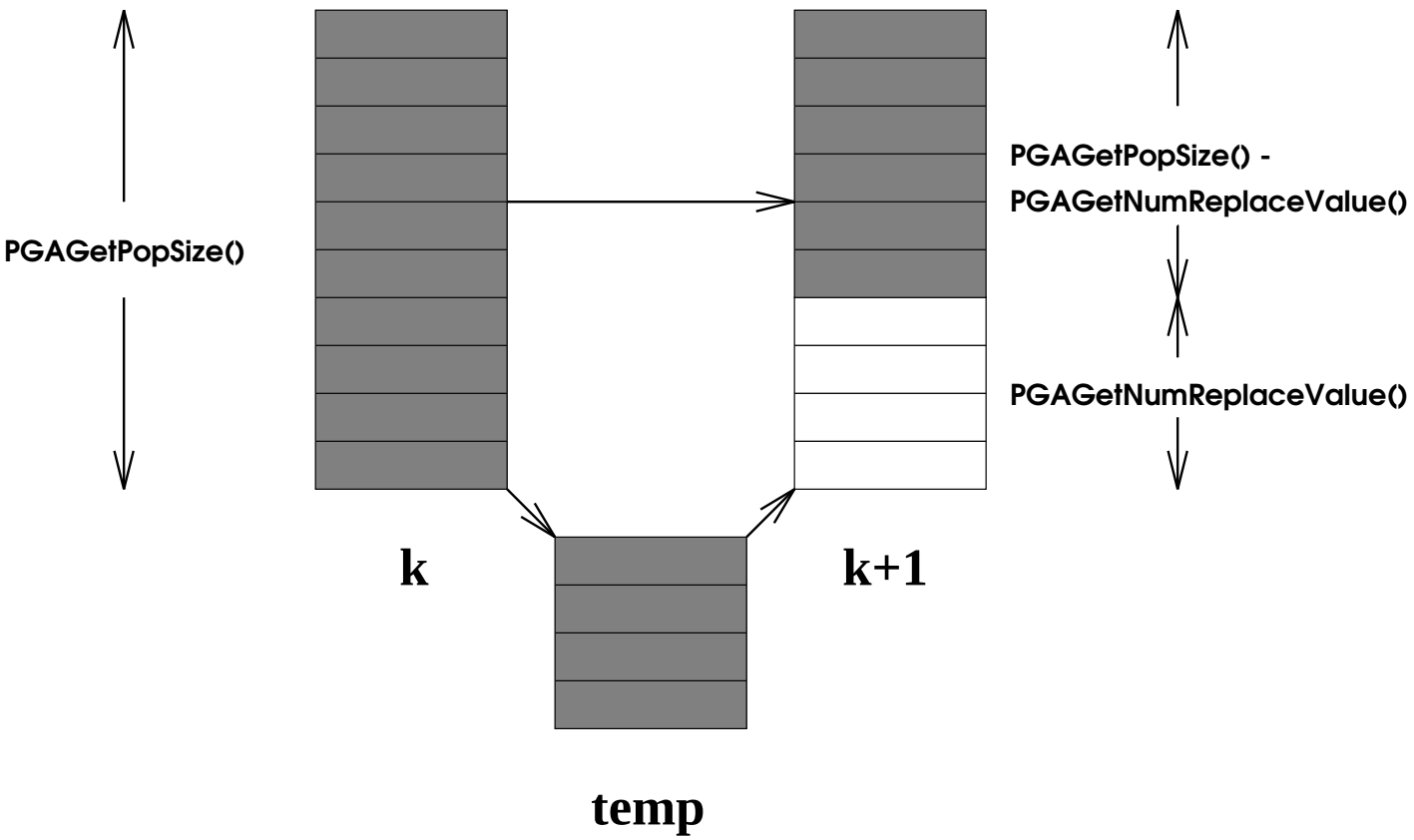

Figure 5.1: Population Replacement

5.3 Stopping Criteria

PGAPack terminates when at least one of the stopping rule(s) specified has been met. The three stopping rules are (1) iteration limit exceeded, (2) population too similar, and (3) no change in the best solution found in a given number of iterations. The default is to stop when the iteration limit (by default, 1000 iterations) is reached.

The choice of stopping rule is set by `PGASetStoppingRuleType`. For example, `PGASetStoppingRuleType (ctx,PGA_STOP_MAXITER)` is the default. Other choices are `PGA_STOP_TOOSIMILAR` and `PGA_STOP_NOCHANGE` for population too similar and no change in the best solution found, respectively. `PGASetStoppingRuleType` may be called more than once. The different stopping rules specified are *or*ed together.

If `PGA_STOP_MAXITER` is one of the stopping rules, `PGASetMaxGAIterValue(ctx,500)` will change the maximum iteration limit to 500. If `PGA_STOP_NOCHANGE` is one of the stopping rules, `PGASetMaxNoChangeValue(ctx,50)` will change from 100 (the default) to 50 the maximum number of iterations in which no change in the best evaluation is allowed before the GA stops. If `PGA_STOP_TOOSIMILAR` is one of the stopping rules, `PGASetMaxSimilarityValue(ctx,99)` will change from 95 to 99 the percentage of the population allowed to have the same evaluation function value before the GA stops.

5.4 Initialization

Strings are either initialized randomly (the default), or set to zero. The choice is specified by setting the second argument of `PGASetRandomInitFlag` to either `PGA_TRUE` or `PGA_FALSE`, respectively. Random initialization depends on the datatype.

If binary-valued strings are used, each gene is set to `1` or `0` with an equal probability. To set the probability of randomly setting a bit to `1` to 0.3, use `PGASetBinaryInitProb(ctx,0.3)`.

For integer-valued strings, the default is to set the strings to a permutation on a range of integers. The default range is $[0, L-1]$, where L is the string length. `PGASetIntegerInitPermute(ctx, 500, 599)` will set the permutation range to $[500, 599]$. The length of the range *must* be the same as the string length.

Alternatively, `PGASetIntegerInitRange` will set each gene to a random value selected uniformly from a specified range. For example, the code

```
stringlen = PGAGetStringLength(ctx);
for(i=0;i<stringlen;i++) {
    low[i]  = 0;
    high[i] = i;
}
PGASetIntegerInitRange(ctx, low, high);
```

will select a value for gene `i` uniformly randomly from the interval `[0,i]`.

If real-valued strings are used, the alleles are set to a value selected uniformly randomly from a specified interval. The interval may be specified with either the `PGASetRealInitRange` or `PGASetRealInitPercent` functions. For example, the code

```
stringlen = PGAGetStringLength(ctx);
for(i=0;i<stringlen;i++) {
    low[i]  = -10.0;
    high[i] = (double) i;
}
PGASetRealInitRange(ctx, low, high);
```

will select a value for allele `i` uniformly randomly from the interval $[-10.0, \mathtt{i}]$. This is the default strategy for initializing real-valued strings. The default interval is $[0, 1.0]$.

`PGASetRealInitPercent` specifies the interval with a median value and percent offset. For example,

```
stringlen = PGAGetStringLength(ctx);
for(i=1;i<=stringlen;i++) {
    median[i]  = (double)  i;
```

```
    percent[i] =          .5;
}
PGASetRealInitPercent(ctx, median, percent);
```

will select a value for allele `i` uniformly randomly from the increasing intervals $[\frac{1}{2}i, \frac{3}{2}i]$. Note that if the median value is zero for some i, than an interval of $[0, 0]$ will be defined.

If character-valued strings are used, `PGASetCharacterInitType(ctx,PGA_CINIT_UPPER)` will set the allele values to uppercase alphabetic characters chosen uniformly randomly. Other options are `PGA_CINIT_LOWER` for lower case letters only (the default) and `PGA_CINIT_MIXED` for mixed case letters, respectively.

5.5 Selection

The selection phase allocates reproductive trials to strings on the basis of their fitness. PGAPack supports four selection schemes: proportional selection, stochastic universal selection, binary tournament selection, and probabilistic binary tournament selection. The choice may be specified by setting the second argument of `PGASetSelectType` to one of `PGA_SELECT_PROPORTIONAL`, `PGA_SELECT_SUS`, `PGA_SELECT_TOURNAMENT`, and `PGA_SELECT_PTOURNAMENT` for proportional, stochastic universal, tournament, and probabilistic tournament selection, respectively. The default is tournament selection. For probabilistic tournament selection, the default probability that the string that wins the tournament is selected is 0.6. It may be set to 0.8, for example, with `PGASetPTournamentProb(ctx, 0.8)`.

5.6 Crossover

The crossover operator takes bits from each parent string and combines them to create child strings. The type of crossover may be specified by setting `PGASetCrossoverType` to `PGA_CROSSOVER_ONEPT`, `PGA_CROSSOVER_TWOPT`, or `PGA_CROSSOVER_UNIFORM` for one-point, two-point, or uniform crossover, respectively. The default is two-point crossover. By default the crossover rate is 0.85. It may be set to 0.6 by `PGASetCrossoverProb(ctx, 0.6)`, for example.

Uniform crossover is parameterized by p_u, the probability of swapping two parent bit values [8]. By default, $p_u = 0.5$. The function call `PGASetUniformCrossoverProb(ctx, 0.7)` will set $p_u = 0.7$.

5.7 Mutation

The mutation *rate* is the probability that a gene will undergo mutation. The mutation rate is independent of the datatype used. The default mutation rate is the reciprocal of the string length. The function call `PGASetMutationProb(ctx,.001)` will set the mutation rate to .001.

The *type* of mutation depends on the data type. For binary-valued strings, mutation is a bit complement operation For character-valued strings, mutation replaces one alphabetic character with another chosen uniformly randomly. The alphabetic characters will be lower, upper, or mixed case depending on how the strings were initialized.

For integer-valued strings, if the strings were initialized to a permutation and gene i is to be mutated, the default mutation operator swaps gene i with a randomly selected gene. If the strings were initialized to a random value from a specified range and gene i is to be mutated, by default gene i will be replaced by a value selected uniformly random from the initialization range.

The mutation operator for integer-valued strings may be changed irrespective of how the strings were initialized. If `PGASetMutationType` is set to `PGA_MUTATION_RANGE`, gene i will be replaced with a value selected uniformly randomly from the initialization range. If the strings were initialized to a permutation, the minimum and maximum values of the permutation define the range. If `PGASetMutationType` is set to `PGA_MUTATION_PERMUTE`, gene i will be swapped with a randomly selected gene. If `PGASetMutationType` is set to `PGA_MUTATION_CONSTANT`, a constant integer value (by default one) will be added (subtracted) to (from) the existing allele value. The constant value may be set to 34, for example, with `PGASetMutationIntegerValue(ctx,34)`.

Three of the four real-valued mutation operators are of the form $v \leftarrow v \pm p \times v$, where v is the existing allele value. They vary by how p is selected. First, if `PGASetMutationType` is set to `PGA_MUTATION_CONSTANT`,

p is the constant value 0.01. It may be set to .02, for example, with `PGASetMutationRealValue(ctx,.02)`. Second, if `PGASetMutationType` is set to `PGA_MUTATION_UNIFORM`, p is selected uniformly from the interval $(0, .1)$. To select p uniformly from the interval $(0, 1)$ set `PGASetMutationRealValue(ctx,1)`. Third, if `PGASetMutationType` is set to `PGA_MUTATION_GAUSSIAN`, p is selected from a Gaussian distribution (this is the default real-valued mutation operator) with mean 0 and standard deviation 0.1. To select p from a Gaussian distribution with mean 0 and standard deviation 0.5 set `PGASetMutationRealValue(ctx,.5)`. Finally, if `PGASetMutationType` is set to `PGA_MUTATION_RANGE`, gene i will be replaced with a value selected uniformly random from the initialization range of that gene.

Some of the integer- and real-valued mutation operators may generate allele values outside the initialization range of that gene. If this happens, by default, the allele value will be reset to the lower (upper) value of the initialization range for that gene. By setting `PGASetMutationBoundedFlag(ctx, PGA_FALSE)` the allele values will *not* be reset if they fall outside of the initialization range.

5.8 Restart

The restart operator reseeds a population from the best string. It does so by seeding the new population with the best string and generating the remainder of the population as mutated variants of the best string.

By default the restart operator is not invoked. Setting `PGASetRestartFlag(ctx,PGA_TRUE)` will cause the restart operator to be invoked. By default PGAPack will restart every 50 iterations. `PGASetRestartFrequencyValue (ctx,100)` will restart every 100 iterations instead. When creating the new strings from the best string an individual allele undergoes mutation with probability 0.5. This can be changed to 0.9 with the function call `PGASetRestartAlleleChangeProb(ctx,0.9)`.

For binary-valued strings the bits are complemented. For integer- and real-valued strings the amount to change is set with `PGASetMutationIntegerValue` and `PGASetMutationRealValue`, respectively. Character-valued strings are changed according to the rules in Section 5.7 for mutating character strings.

5.9 String Evaluation and Fitness

In a genetic algorithm each string is assigned a nonnegative, real-valued *fitness*. This is a measure, relative to the rest of the population, of how well that string satisfies a problem-specific metric. In PGAPack calculating a string's fitness is a two-step process. First, the *user* supplies a real-valued evaluation (sometimes called the raw fitness) of each string. Second, this value is mapped to a fitness value.

It is the user's responsibility to supply a function to evaluate an individual string. As discussed in Section 5.1, the name of this function is specified as the second argument to `PGARun`. The calling sequence for this function (which we call `evaluate` in the rest of this section, but may have any name) *must* follow the format given here. In C the format is

```
double evaluate (PGAContext *ctx, int p, int pop);
```

and in Fortran

```
double precision function evaluate (ctx, p, pop)
integer ctx, p, pop
```

The function `evaluate` will be called by `PGARun` whenever a string evaluation is required. `p` is the index of the string in population `pop` that will be evaluated. The correct values of `p` and `pop` will be passed to the evaluation function by `PGARun`. (If `PGARun` is not used, `PGAEvaluate` must be. See Chapter 6.) As shown below, `p` and `pop` are used for reading (and sometimes writing) allele values Sample evaluation functions are shown in Figures 3.1 and 3.2, and online in the `./examples` directory.

Traditionally, genetic algorithms assume fitness values are nonnegative and monotonically increasing the more fit a string is. The user's evaluation of a string, however, may reflect a minimization problem and/or be negative. Therefore, the user's *evaluation value* is mapped to a nonnegative and monotonically increasing *fitness value*. First, all evaluations are mapped to positive values (if any were negative). Next, these values are translated to a maximization problem (if the direction of optimization specified was minimization).

Finally, these values are mapped to a fitness value by using the identity (the default), linear ranking, or linear normalization, The choice of fitness mapping may be set with the function `PGASetFitnessType`. The second argument must be one of `PGA_FITNESS_RAW`, `PGA_FITNESS_RANKING`, or `PGA_FITNESS_NORMAL`, for the identity, linear ranking, or linear normalization, respectively.

A *linear rank* fitness function [2, 10] is given by

$$Min + (Max - Min)\frac{\texttt{rank(p)} - 1}{N - 1}, \tag{5.1}$$

where `rank(p)` is the index of string `p` in a list sorted in order of decreasing evaluation function value, and N is the population size. Ranking requires that $1 \leq Max \leq 2$, and $Min + Max = 2$. The default value for Max is 1.2. It may be set to 1.1 with `PGASetMaxFitnessRank(ctx,1.1)`.

In *linear normalization* the fitness function is given by

$$K - (\texttt{rank(p)} * C), \tag{5.2}$$

where K and C are the constants $\sigma * N$ and σ, where σ is the standard deviation of the user's evaluation function values after they have been transformed to positive values for a maximization problem.

If the direction of optimization is minimization, the values are remapped for maximization. The function call `PGASetFitnessMinType(ctx,PGA_FITNESSMIN_CMAX)` will remap by subtracting the worst evaluation value from each evaluation value (this is the default). The worst evaluation value is multiplied by 1.01 before the subtraction so that the worst string has a nonzero fitness. The function call `PGASetFitnessCmaxValue(ctx, 1.2)` will change the multiplier to 1.2 Alternatively, if `PGA_FITNESSMIN_RECIPROCAL` is specified the remapping is done by using the reciprocal of the evaluation function.

5.10 Accessing Allele Values

For each of the native data types, PGAPack provides a matched pair of functions that allow the user to read or write (change) any allele value. If the data type is `PGA_DATATYPE_BINARY`

```
int bit;
bit = PGAGetBinaryAllele (ctx, p, pop, i);
```

will assign to `bit` the binary value of the `i`th gene in string `p` in population `pop`. To set the `i`th gene in string `p` in population `pop` to `1`, use

```
PGASetBinaryAllele(ctx, p, pop, i, 1);
```

If the data type is `PGA_DATATYPE_INTEGER`

```
int k;
k = PGAGetIntegerAllele (ctx, p, pop, i);
```

will assign to `k` the integer value of the `i`th gene in string `p` in population `pop`. To set the `i`th gene in string `p` in population `pop` to 34, use

```
PGASetIntegerAllele(ctx, p, pop, i, 1, 34);
```

If the data type is `PGA_DATATYPE_REAL`

```
double x;
x = PGAGetRealAllele (ctx, p, pop, i);
```

will assign to `x` the real value of the `i`th gene in string `p` in population `pop`. To set the `i`th gene in string `p` in population `pop` to 123.456, use

```
PGASetRealAllele(ctx, p, pop, i, 1, 123.456);
```

If the data type is `PGA_DATATYPE_CHARACTER`

```
char c;
c = PGAGetCharacterAllele (ctx, p, pop, i);
```

will assign to `c` the character value of the `i`th gene in string `p` in population `pop`. To set the `i`th gene in string `p` in population `pop` to "Z", use

```
PGASetCharacterAllele(ctx, p, pop, i, 1, 'Z');
```

5.10.1 Representing an Integer with a Binary String

A binary string may be used to represent an integer by *decoding* the bits into an integer value. In a binary coded decimal (BCD) representation, a binary string is decoded into an integer $k \in [0, 2^N - 1]$ according to

$$k = \sum_{i=1}^{N} b_i 2^{i-1}, \tag{5.3}$$

where N is the string length, and b_i the value of the ith bit. For example, to decode the integer `k` from the ten bits in bit positions 20–29, use

```
int k
k = PGAGetIntegerFromBinary(ctx,p,pop,20,29);
```

The function `PGAEncodeIntegerAsBinary` will encode an integer as a binary string. For example, to encode the integer 564 as a 12-bit binary string[2] in the substring defined by bits 12–23, use

```
PGAEncodeIntegerAsBinary(ctx,p,pop, 12, 23, 564);
```

In a BCD representation, two numbers that are contiguous in their decimal representations may be far from each other in their binary representations. For example, 7 and 8 are consecutive integers, yet their 4-bit binary representations, `0111` and `1000`, differ in the maximum number of bit positions.[3] *Gray codes* define a different mapping of binary strings to integer values from that given by Eq. (5.3) and may alternatively be used for representing integer (or real, see below) values in a binary string. The second and third columns in Table 5.1 show how the integers 0–7 are mapped to Eq. (5.3) and to the *binary reflected* Gray code (the most commonly used Gray code sequence), respectively. In the binary reflected Gray code sequence, the binary representations of consecutive integers differ by only one bit (a Hamming distance of one).

To decode the integer `k` from a binary reflected Gray code interpretation of the binary string, use

```
k = PGAGetIntegerFromGrayCode(ctx,p,pop,20,29);
```

To encode 564 as a 12-bit binary string in the substring defined by bits 12–23 using a Gray code, use

```
PGAEncodeIntegerAsGaryCode(ctx,p,pop, 12, 23, 564);
```

5.10.2 Representing a Real Value with a Binary String

A binary string may also be used to represent a real value. The decoding of a binary string to a real-value is a two-step process. First, the binary string is decoded into an integer as described in Section 5.10.1. Next, the integer is mapped from the discrete interval $[0, 2^N - 1]$ to the real interval $[L, U]$ by using the formula

$$x = (k - a) \times (U - L)/(b - a) + L$$

(and generalizing $[0, 2^N - 1]$ to $[a, b]$). For example, to decode the `double x` from the 20 bits given by the binary string stored in bit positions 10–29 onto the interval $[-10.0, 20.0]$, use

```
x = PGAGetRealFromBinary(ctx,p,pop,10,29,-10.0,20.0);
```

[2] Even though only ten bits are necessary to encode 564, the user may want to allow the GA any value between $[0, 4095]$, hence the twelve bits.

[3] Technically, this is known as a Hamming cliff.

Table 5.1: Binary and Gray Codes

k	Eq. (5.3)	Gray code
0	000	000
1	001	001
2	010	011
3	011	010
4	100	110
5	101	111
6	110	101
7	111	100

To encode -18.3 on the interval $[-50.0, 50.0]$ using a 20-bit BCD binary string, use

```
PGAEncodeRealAsBinary(ctx,p,pop,0,19,-50.0,50.0,-18.3);
```

The functions `PGAGetRealFromGrayCode` and `PGAEncodeRealAsGrayCode` provide similar functionality for Gray-coded strings.

5.10.3 Example

As an example, suppose the user has a real-valued function f of three real variables x_1, x_2, and x_3. Further, the variables are constrained as follows.

$$-10 \leq x_1 \leq 0$$

$$0 \leq x_2 \leq 10$$

$$-10 \leq x_3 \leq 10$$

The user wishes to use 10 bits for the binary representation of x_1 and x_2, and 20 bits for the binary representation of x_3 (perhaps for higher accuracy), and a Gray code encoding. This may be done as follows.

```
#include "pgapack.h"
double grayfunc (PGAContext *ctx, int p, int pop);
double f        (double x1, double x2, double x3);
int main(int argc, char **argv)
{
    PGAContext *ctx;
    ctx = PGACreate (&argc, argv, PGA_DATATYPE_BINARY, 40, PGA_MINIMIZE);
    PGASetUp        (ctx                                               );
    PGARun          (ctx, grayfunc                                     );
    PGADestroy      (ctx                                               );
    return;
}

double grayfunc (PGAContext *ctx, int p, int pop)
{
    double x1, x2, x3, v;
    x1 =  PGAGetRealFromGrayCode (ctx, p, pop,  0,  9, -10.,  0.);
    x2 =  PGAGetRealFromGrayCode (ctx, p, pop, 10, 19,   0., 10.);
    x3 =  PGAGetRealFromGrayCode (ctx, p, pop, 20, 39, -10., 10.);
    v  =  f(x1,x2,x3);
    return(v);
}
```

In Fortran, the bit indices would be 1–10, 11–20, and 21–40, respectively. The number of bits allocated for the binary representation determines the accuracy with which the real value can be calculated. Note in this

example the function `f` *need not be modified*; the function `grayfunc` is used as a "wrapper" to get variable values out of the GA and return the value calculated by `f`.

5.11 Report Options

`PGASetPrintFrequencyValue(ctx,40)` will print population statistics every 40 iterations. The default is every ten iterations. The best evaluation is *always* printed. To print additional statistics, set the second argument of the function `PGASetPrintOptions` to `PGA_REPORT_ONLINE`, `PGA_REPORT_OFFLINE`, `PGA_REPORT_WORST`, `PGA_REPORT_AVERAGE`, `PGA_REPORT_HAMMING`, or `PGA_REPORT_STRING` to print the online analysis, offline analysis, worst evaluation, average evaluation, Hamming distance, or string itself, respectively. `PGASetPrintOptions` may be called multiple times to specify multiple print options.

5.12 Utility Functions

5.12.1 Random Numbers

By default, PGAPack will seed its random number generator by using a value from the system clock. Therefore, each time PGAPack is run, a unique sequence of random numbers will be used. For debugging or reproducibility purposes, however, the user may wish to use the same sequence of random numbers each time. This may be done using the function `PGASetRandomSeed` to initialize the random number generator with the same seed each time, for example, `PGASetRandomSeed(ctx,1)`.

`PGARandom01(ctx,0)` will return a random number generated uniformly on $[0, 1]$. If the second argument is not `0`, it will be used to reseed the random number sequence. `PGARandomFlip` flips a biased coin. For example, `PGARandomFlip(ctx,.7)` will return `PGA_TRUE` approximately 70% of the time. `PGARandomInterval(-10,30)` will return an integer value generated uniformly on $[-10, 30]$. `PGARandomUniform (ctx,-50.,50.)` will return a real value generated uniformly randomly on the interval [-50,50]. `PGARandomGaussian (ctx,0.,1.)` will return a real value generated from a Gaussian distribution with mean zero and standard deviation one.

5.12.2 Print Functions

`PGAPrintPopulation(ctx,stdout,pop)` will print the evaluation function value, fitness value, and string for each member of population `pop` to `stdout`. This function may not be called until *after* `PGASetUp` has been called. `PGAPrintContextVariable(ctx,stdout)` will print the value of all fields in the context variable to `stdout`. `PGAPrintIndividual(ctx,stdout,p,pop)` will print the evaluation function value, fitness value, and string of individual `p` in population `pop` to `stdout`. `PGAPrintString(ctx,stdout,p,pop)` will print the string of individual `p` in population `pop` to `stdout`. `PGAPrintVersionNumber(ctx)` will print the PGAPack version number.

5.12.3 Miscellaneous

`PGAGetGAIterValue(ctx)` will return the current iteration of the GA. `PGAGetBestIndex(ctx,pop)` (`PGAGetWorstIndex`) will return the index of the most (least) fit member of population `pop`.

`PGAUpdateOffline(ctx,pop)` (`PGAUpdateOnline`) will update the offline (online) analysis based on the new generation's results. `PGAHammingDistance(ctx,pop)` returns a `double`, which is the average Hamming distance between the *binary* strings in population `pop`. The function call

```
PGAError(ctx, "popindex=", PGA_FATAL, PGA_INT, (void *)&popindex)
```

will print the message "popindex=-1" (assuming the value of `popindex` is -1) and then exit PGAPack. If the third argument had been `PGA_WARNING` instead, execution would have continued. In addition to `PGA_INT`, valid data types are `PGA_DOUBLE`, `PGA_CHAR`, and `PGA_VOID`.

5.13 Command-Line Arguments

PGAPack provides several command-line arguments. These are only available to C programs, although in some cases both C and Fortran programs can achieve the equivalent functionality with function calls. For example, `PGAUsage(ctx)` provides the same functionality as the `-pgahelp` command line option. See Chapter 12 for the function call equivalents.

```
-pgahelp            get this message
-pgahelp   debug    list of debug options
-pgadbg   <level>   set debug option
-pgadebug <level>   set debug option
-pgaversion         Print current PGAPack version number, parallel or
                    sequential, and debug or optimized
```

Chapter 6

Explicit Usage

This chapter discusses how the user may obtain greater control over the steps of the GA by *not* using the `PGARun` command, but instead calling the data-structure-neutral functions directly. One ramification of this is that the `PGARun` interface no longer masks some of the differences between parallel and sequential execution. The examples in this chapter are written for sequential execution *only*. Chapter 10 shows how they may be executed in parallel.

6.1 Notation

To understand the calling sequences of the functions discussed in this chapter, one must know of the *existence* of certain data structures and the user interface for accessing them. It is *not* necessary to know how these data structures are implemented, since that is hidden by the user interface to PGAPack.

PGAPack maintains two populations: an *old* one and a *new* one. The size of each population is the value returned by `PGAGetPopSize`. In addition, each population contains two temporary working locations. The string length is the value specified to `PGACreate` and returned by `PGAGetStringLength`.

Formally, string p in population *pop* is referred to by the 2-tuple `(p,pop)` and the value of gene i in that string by the 3-tuple `(i,p,pop)`. In PGAPack, `pop` *must* be one of the two symbolic constants `PGA_OLDPOP` or `PGA_NEWPOP` to refer to the old or new population, respectively. At the end of each GA iteration, the function `PGAUpdateGeneration` makes sure these symbolic constants are remapped to the correct population. The string index `p` must be either an integer between 0 and $P-1$ (or 1 and P in Fortran) or one of the symbolic constants `PGA_TEMP1` or `PGA_TEMP2`, to reference one of the two temporary locations, respectively.

6.2 Simple Sequential Example

The example in Figure 6.1 is a complete PGAPack program that does *not* use `PGARun`. It is an alternative way to write the main program for the Maxbit example of Section 3.1. We refer to it as a simple example because it uses `PGARunMutationAndCrossover` to encapsulate the recombination step. The `PGACreate` and `PGASetUp` functions were discussed in the last chapter. `PGASetUp` creates and randomly initializes the initial population. This population, referred to initially by the symbolic constant `PGA_OLDPOP`, is evaluated by the `PGAEvaluate` function. The third argument to `PGAEvaluate` is the name of the user's evaluation function. The function prototype for `evaluate` must be as shown in Figure 6.1 and discussed earlier in Sections 5.1 and 5.9. The `PGAFitness` function maps the user's evaluation function values into fitness values.

The `while` loop runs the genetic algorithm. `PGADone` returns `PGA_TRUE` if any of the specified stopping criteria have been met, otherwise `PGA_FALSE`. `PGASelect` performs selection on population `PGA_OLDPOP`. `PGARunMutationAndCrossover` uses the selected strings to create the new population by applying the crossover and mutation operators. `PGAEvaluate` and `PGAFitness` evaluate and map to fitness values the newly created population. `PGAUpdateGeneration` updates the GA iteration count and resets several important internal arrays (don't forget to call it!). `PGAPrintReport` writes out genetic algorithm statistics

```
#include "pgapack.h"
double evaluate (PGAContext *ctx, int p, int pop);

int main(int argc, char **argv)
{
    PGAContext *ctx;

    ctx = PGACreate(&argc, argv, PGA_DATATYPE_BINARY, 100, PGA_MAXIMIZE);
    PGASetUp   (ctx);
    PGAEvaluate(ctx, PGA_OLDPOP, evaluate, NULL);
    PGAFitness (ctx, PGA_OLDPOP);

    while(!PGADone(ctx, NULL)) {
        PGASelect                 (ctx, PGA_OLDPOP);
        PGARunMutationAndCrossover(ctx, PGA_OLDPOP, PGA_NEWPOP);
        PGAEvaluate               (ctx, PGA_NEWPOP, evaluate, NULL);
        PGAFitness                (ctx, PGA_NEWPOP);
        PGAUpdateGeneration       (ctx, NULL);
        PGAPrintReport            (ctx, stdout, PGA_OLDPOP);
    }
    PGADestroy(ctx);
    return(0);
}
```

Figure 6.1: Simple Example of Explicit Usage

according to the report options specified. Note that the argument to `PGAPrintReport` is the old population, since after `PGAUpdateGeneration` is called, the newly created population is in `PGA_OLDPOP`. Finally, `PGADestroy` releases any memory allocated by PGAPack when execution is complete.

The functions `PGADone`, `PGAUpdateGeneration`, and `PGAEvaluate` take an MPI communicator (see Appendix C and Chapter 10) as an argument. For *sequential* execution the value `NULL` should be specified for this argument. A parallel, or sequential *and* parallel, version of this example is given in Section 10.2.

6.3 Complex Example

The primary difference between the "complex" example in Figure 6.2 and the "simple" example in Figure 6.1 is that the steps encapsulated by `PGARunMutationAndCrossover` have been written out explicitly. The function `PGASortPop` sorts a population according to the criteria specified by `PGASetPopReplacementType` (Section 5.2). The sorted indices are accessed via `PGAGetSortedPopIndex`. In the example, the five lines that follow `PGASortPop` copy the strings that are not created by recombination from the old population to the new population.

The `while` loop that follows creates the remainder of the new population. `PGASelectNextIndex` returns the indices of the strings selected by `PGASelect`. `PGARandomFlip` flips a coin biased by the crossover probability to determine whether the selected strings should undergo crossover and mutation or should be copied directly into the new population. `PGACrossover` uses the parent strings `m1` and `m2` from population `PGA_OLDPOP` to create two child strings in the temporary locations `PGA_TEMP1` and `PGA_TEMP2` in `PGA_NEWPOP` population.

`PGAMutate` mutates the child strings and `PGACopyIndividual`, then copies them into the new population. If the strings do not undergo crossover and mutation, they are copied into the new population unchanged. The rest of the steps are the same as those in Figure 6.1, *except* that for illustrative purposes we call `PGAPrintReport` *before* `PGAUpdateGeneration`. In that case we use population `PGA_NEWPOP` as the population pointer.

```
#include "pgapack.h"
double evaluate(PGAContext *ctx, int p, int pop);

int main(int argc, char **argv)
{
    PGAContext *ctx;
    int i, j, n, m1, m2, popsize, numreplace;
    double probcross;

    ctx = PGACreate(&argc, argv, PGA_DATATYPE_BINARY, 100, PGA_MAXIMIZE);
    PGASetUp(ctx);
    probcross  = PGAGetCrossoverProb(ctx);
    popsize    = PGAGetPopSize(ctx);
    numreplace = PGAGetNumReplaceValue(ctx);
    PGAEvaluate(ctx, PGA_OLDPOP, evaluate, NULL);
    PGAFitness (ctx, PGA_OLDPOP          );

    while(!PGADone(ctx, NULL)) {
        PGASelect (ctx, PGA_OLDPOP);
        PGASortPop(ctx, PGA_OLDPOP);
        n = popsize - numreplace;
        for ( i=0; i < n; i++ ) {
            j = PGAGetSortedPopIndex(ctx, i);
            PGACopyIndividual(ctx, j, PGA_OLDPOP, i, PGA_NEWPOP);
        }
        while (n < popsize) {
            m1 = PGASelectNextIndex(ctx);
            m2 = PGASelectNextIndex(ctx);
            if(PGARandomFlip(ctx, probcross)) {
                PGACrossover(ctx, m1, m2, PGA_OLDPOP, PGA_TEMP1, PGA_TEMP2, PGA_NEWPOP);
                PGAMutate (ctx,PGA_TEMP1,PGA_NEWPOP);
                PGAMutate (ctx,PGA_TEMP2,PGA_NEWPOP);
                PGACopyIndividual (ctx,PGA_TEMP1,PGA_NEWPOP,n,  PGA_NEWPOP);
                PGACopyIndividual (ctx,PGA_TEMP2,PGA_NEWPOP,n+1,PGA_NEWPOP);
                n += 2;
            }
            else {
                PGACopyIndividual (ctx, m1, PGA_OLDPOP, n,   PGA_NEWPOP);
                PGACopyIndividual (ctx, m2, PGA_OLDPOP, n+1, PGA_NEWPOP);
                n += 2;
            }
        }
        PGAEvaluate(ctx, PGA_NEWPOP, evaluate, NULL);
        PGAFitness (ctx, PGA_NEWPOP);
        PGAPrintReport(ctx, stdout, PGA_NEWPOP);
        PGAUpdateGeneration(ctx, NULL);
    }
    PGADestroy(ctx);
    return 0;
}
```

Figure 6.2: Example of Explicit Usage

6.4 Explicit PGAPack Functions

This section briefly discusses other functions not shown in the previous examples or discussed in Chapter 5. Additional information about these and other PGAPack functions is contained in Appendix B (function bindings) and the `./examples` directory.

`PGARunMutationAndCrossover` and `PGARunMutationOrCrossover` perform the recombination step. The former applies mutation to strings that undergo crossover. The latter applies only mutation to strings that did not undergo crossover.

The restart operator described earlier in Section 5.8 can be invoked explicitly with `PGARestart(ctx, oldpop, newpop)`, where the best string from population `oldpop` is used to initialize population `newpop`.

`PGADuplicate(ctx,p,PGA_OLDPOP,PGA_NEWPOP,20)` returns `PGA_TRUE` if string `p` in population `PGA_OLDPOP` is a duplicate of any of the first 20 strings in population `PGA_NEWPOP`. `PGAChange(ctx, p, PGA_OLDPOP)` repeatedly applies the mutation operator to string `p` in population `PGA_OLDPOP` until at least one mutation has occurred.

In PGAPack three values are associated with each string: (1) the user's evaluation function value, (2) a Boolean flag to indicate whether the evaluation function value is up to date with respect to the actual string, and (3) the fitness value. If `PGARun` is not used, the user must manage these values explicitly.

`PGAEvaluate(ctx, PGA_NEWPOP, evaluate, comm)` will execute the user's evaluation function, `evaluate`, on each string in population `PGA_NEWPOP` that has changed (for example, from crossover) since its last evaluation. `PGAEvaluate` will set both the evaluation function value and associated Boolean flag automatically. The argument `comm` is an MPI communicator. Valid values are `NULL` for an explicitly sequential example, or any valid MPI communicator. Depending on the number of processes specified when the program was invoked, and the value of the `comm` argument, `PGAEvaluate` may be run with one or more processes. See Chapter 10 for further discussion.

`PGAFitness` will calculate the population fitness values from the evaluation function values. It is an error to call `PGAFitness` if *all* the evaluation function values are not up to date.

These same three values may be read also. `PGAGetEvaluation(ctx, p, PGA_OLDPOP)` returns the evaluation function value. `PGAGetEvaluationUpToDateFlag(ctx, p, PGA_OLDPOP)` returns `PGA_TRUE` or `PGA_FALSE` to indicate whether the evaluation is up to date with the actual string or not, respectively. If PGAPack was compiled for debugging,`PGAGetEvaluation` will print a warning message if the evaluation is not up to date. `PGAGetFitness(ctx, p, PGA_OLDPOP)` returns the fitness value.

At times, (e.g., applying a hill-climbing function) the user may need to explicitly set the evaluation function value and associated Boolean flag (fitness values can be calculated *only* by calling `PGAFitness`). `PGASetEvaluation(ctx, p, PGA_OLDPOP, 123.4)` will set the evaluation function value to 123.4 and the associated Boolean flag to `PGA_TRUE`. The Boolean flag may be set independently with `PGASetEvaluationUpToDateFlag`. For example, `PGASetEvaluationUpToDateFlag (ctx, p, PGA_OLDPOP, PGA_FALSE)` sets the status of the Boolean flag of string `p` in population `PGA_OLDPOP` to out of date.

`PGAMean(ctx, a, n)` returns the mean of the `n` values in array `a`. `PGAStddev(ctx, a, n, mean)` returns the standard deviation of the `n` values in array `a` whose mean is `mean`. `PGARank(ctx, p, order, n)` returns an index that is the rank of string p as given by the sorted array `order` of length `n`.

`PGAGetPrintFrequency(ctx)` returns the frequency with which GA statistics are reported. `PGAGetWorstIndex (ctx, PGA_OLDPOP)` returns the index of the string in population `PGA_OLDPOP` with the worst evaluation function value. `PGAGetBestIndex(ctx, PGA_OLDPOP)` returns the index of the string in population `PGA_OLDPOP` with the best evaluation function value.

Chapter 7

Custom Usage: Native Data Types

This chapter discusses how PGAPack may be extended by replacing some of the standard PGAPack functions with user-defined functions for use with one of PGAPack's four *native* data types. This can be done from both C and Fortran.

7.1 Basics

In PGAPack, high-level (data-structure-neutral) functions call data-structure-specific functions that correspond to the data type used. The implementation uses function pointers that, by default, are set to the correct values for the datatype used. The user may change these defaults and set the function pointers to execute their functions instead. The functions the user can substitute for are initialization, crossover, mutation, checking for duplicate strings, string printing, termination criteria, and a generic function called at the end of each GA iteration.

The function call `PGASetUserFunction(ctx, PGA_USERFUNCTION_MUTATION, mymute)` will cause PGAPack to execute the function `mymute` whenever the mutation operator is called. Table 7.1 is a list of functions that can be customized for use with a native datatype. The first column describes the functionality, and the second column the symbolic constant for use with `PGASetUserFunction`. The calling sequence for these functions is fixed and *must* follow the function prototypes in Table 7.2. The files `./examples/templates/uf_native.c` and `./examples/templates/uf_native.f` contain template routines for these functions. A specific example is given below.

Checking the termination criteria requires some discussion. The function `PGADone` will *either* check to see if the standard stopping criteria (see Section 5.3) have been met, or call the user function specified by `PGA_USERFUNCTION_STOPCOND`. If you wish to have the user function check for the standard stopping criteria in addition to whatever else it does, it should call `PGACheckStoppingConditions(ctx)`. Do *not* call `PGADone` as this will cause an infinite loop to occur. Note that in a parallel program `PGACheckStoppingConditions` should only be called by the master process (see Chapter 10).

The end of generation function (which is null by default) may be used for gathering statistics about the GA, displaying custom output, etc. This function is called after all generational computation is complete, but

Table 7.1: Customizeable Functions: Native Data Types

Functionality	Symbolic Constant
Initialization	`PGA_USERFUNCTION_INITSTRING`
Crossover	`PGA_USERFUNCTION_CROSSOVER`
Mutation	`PGA_USERFUNCTION_MUTATION`
Duplicate Checking	`PGA_USERFUNCTION_DUPLICATE`
String Printing	`PGA_USERFUNCTION_PRINTSTRING`
Termination Criteria	`PGA_USERFUNCTION_STOPCOND`
End of generation	`PGA_USERFUNCTION_ENDOFGEN`

Table 7.2: Calling Sequences for Customizable Functions

Symbolic Constant	Return	Function Prototype
PGA_USERFUNCTION_INITSTRING	void	(PGAContext*, int, int)
PGA_USERFUNCTION_CROSSOVER	void	(PGAContext*, int, int, int, int, int, int)
PGA_USERFUNCTION_MUTATION	int	(PGAContext*, int, int, double)
PGA_USERFUNCTION_DUPLICATE	int	(PGAContext*, int, int, int, int)
PGA_USERFUNCTION_PRINTSTRING	void	(PGAContext*, FILE *, int, int)
PGA_USERFUNCTION_STOPCOND	int	(PGAContext*)
PGA_USERFUNCTION_ENDOFGEN	void	(PGAContext*)

before the population pointers (PGA_NEWPOP, PGA_OLDPOP) have been switched and the standard PGAPack output printed. Therefore, be sure to use PGA_NEWPOP as the population pointer. There is no mechanism for suppressing the standard PGAPack generational output.

7.2 Example Problem: C

The example problem in Figure 7.1 is to maximize $\sum_{j=1}^{L} x_j$ with $1 \leq x_j \leq L$, where L is the string length. The optimal solution to this problem, L^2, is achieved by setting each x_j to L. The files for this example, ./examples/maxint.c and ./examples/maxint.f, contain template routines for these functions.

The example shows the use of a custom mutation function with an integer data type. The PGASetUserFunction function specifies that this function, MyMutation, will be called when the mutation operator is applied, rather than the default mutation operator. MyMutation generates a random integer on the interval $[1, L]$.

7.3 Example Problem: Fortran

Figure 7.2 is the same example as in Figure 7.1 written in Fortran.

```
#include <pgapack.h>

double evaluate (PGAContext *ctx, int p, int pop);
int myMutation  (PGAContext *ctx, int p, int pop, double pm);

int main( int argc, char **argv )
{
     PGAContext *ctx;
     int i, maxiter;
     ctx = PGACreate (&argc, argv, PGA_DATATYPE_INTEGER, 10, PGA_MAXIMIZE);
     PGASetUserFunction      (ctx, PGA_USERFUNCTION_MUTATION, myMutation);
     PGASetIntegerInitPermute(ctx, 1, 10);
     PGASetUp                (ctx);
     PGARun                  (ctx, evaluate);
     PGADestroy              (ctx);
     return(0);
}
int myMutation(PGAContext *ctx, int p, int pop, double pm)
{
    int stringlen, i, k, count = 0;
    stringlen = PGAGetStringLength(ctx);
    for (i = 0; i < stringlen; i++)
    if (PGARandomFlip(ctx, pm)) {
        k = PGARandomInterval(ctx, 1, stringlen);
        PGASetIntegerAllele(ctx, p, pop, i, k);
        count++;
    }
    return ((double) count);
}
double evaluate(PGAContext *ctx, int p, int pop)
{
     int stringlen, i, sum = 0;
     stringlen = PGAGetStringLength(ctx);
     for (i = 0; i < stringlen; i++)
         sum += PGAGetIntegerAllele(ctx, p, pop, i);
     return ((double)sum);
}
```

Figure 7.1: PGAPack C Example Using Custom Mutation Operator

```
 include 'pgapackf.h'
 include 'mpif.h'
 double precision evaluate
 integer          myMutation
 external         evaluate, myMutation
 integer ctx, i, maxiter, ierror
 call MPI_Init(ierror)
 ctx = PGACreate (PGA_DATATYPE_INTEGER, 10, PGA_MAXIMIZE)
 call PGASetUserFunction      (ctx, PGA_USERFUNCTION_MUTATION,
&     myMutation)
 call PGASetIntegerInitPermute(ctx, 1, 10);
 call PGASetUp                (ctx);
 call PGARun                  (ctx, evaluate);
 call PGADestroy              (ctx);
 call MPI_Finalize(ierror)
 stop
 end

 integer function myMutation(ctx, p, pop, pm)
 include          'pgapackf.h'
 integer           ctx, p, pop
 double precision  pm
 integer           stringlen, i, k, count
 count = 0
 stringlen = PGAGetStringLength(ctx)
 do i=0, stringlen
    if (PGARandomFlip(ctx, pm) .eq. PGA_TRUE) then
       k = PGARandomInterval(ctx, 1, stringlen)
       call PGASetIntegerAllele(ctx, p, pop, i, k)
       count = count + 1
    endif
 enddo
 myMutation = count
 return
 end

 double precision function evaluate(ctx, p, pop)
 include      'pgapackf.h'
 integer ctx, p, pop
 integer       stringlen, i, sum
 sum = 0
 stringlen = PGAGetStringLength(ctx)
 do i=0, stringlen
    sum = sum + PGAGetIntegerAllele(ctx, p, pop, i)
 enddo
 evaluate = sum
 return
 end
```

Figure 7.2: PGAPack Fortran Example Using Custom Mutation Operator

Chapter 8

Custom Usage: New Data Types

This chapter discusses how PGAPack may be extended by defining a new data type. Defining a new data type may be done only in C programs.

8.1 Basics

To create a new data type, you must (1) specify `PGA_DATATYPE_USER` for the datatype in the `PGACreate` call and (2) for *each* entry in Table 8.1, call `PGASetUserFunction` to specify the function that will perform the given operation on the new data type. If the data type is `PGA_DATATYPE_USER`, the string length specified to `PGACreate` can be whatever the user desires. It will be returned by `PGAGetStringLength` but is not otherwise used in the data-structure-neutral functions of PGAPack.

The calling sequences for the functions in Table 8.1 are given in Table 8.2. The file `./examples/templates/uf_new.c` contains template routines for these functions.

While PGAPack requires that the user supply all the functions in Table 8.1, your program may not require the functionality of all of them. For example, the user really does not need to write a function to pack the strings for message-passing unless a parallel version of PGAPack is being used. In these cases, we suggest that the user supply a stub function; i.e., a function with the correct calling sequence but no functionality.

8.2 Example Problem

This example illustrates use of a user-defined structure as the new data type. The problem is one of molecular docking where one protein molecule (the ligand) is to be docked into a second, target protein molecule. Figure 8.1 contains the function prototypes for each function that will operate on the new datatype, the definition of the user's structure (`ligand`), and the main program.

The first three `doubles` of the array `t` in structure `ligand` represent the translation of the ligand molecule in the x-, y-, and z-axes, respectively. The last three `doubles` in the array `t` represent the rotation of the

Table 8.1: Functions Required for New Data Types

Functionality	Symbolic Constant
Memory allocation	`PGA_USERFUNCTION_CREATESTRING`
String packing	`PGA_USERFUNCTION_BUILDDATATYPE`
Mutation	`PGA_USERFUNCTION_MUTATION`
Crossover	`PGA_USERFUNCTION_CROSSOVER`
String printing	`PGA_USERFUNCTION_PRINTSTRING`
String copying	`PGA_USERFUNCTION_COPYSTRING`
Duplicate checking	`PGA_USERFUNCTION_DUPLICATE`

Table 8.2: Calling Sequences for New Data Type Functions

Symbolic Constant	Return	Function Prototype
PGA_USERFUNCTION_CREATESTRING	void	(PGAContext*, int, int, int)
PGA_USERFUNCTION_BUILDDATATYPE	int	(PGAContext*, int, int)
PGA_USERFUNCTION_MUTATION	int	(PGAContext*, int, int, double)
PGA_USERFUNCTION_CROSSOVER	void	(PGAContext*, int, int, int, int, int, int)
PGA_USERFUNCTION_PRINTSTRING	void	(PGAContext*, FILE *, int, int)
PGA_USERFUNCTION_COPYSTRING	int	(PGAContext*, int, int, int, int)
PGA_USERFUNCTION_DUPLICATE	int	(PGAContext*, int, int, int, int)

```
#include <pgapack.h>

double       energy           (double *, int *);
double       Evaluate         (PGAContext *, int, int);
void         CreateString     (PGAContext *, int, int, int);
int          Mutation         (PGAContext *, int, int, double);
void         Crossover        (PGAContext *, int, int, int, int, int, int);
void         WriteString      (PGAContext *, FILE *, int, int);
void         CopyString       (PGAContext *, int, int, int, int);
int          DuplicateString  (PGAContext *, int, int, int, int);
MPI_Datatype BuildDT          (PGAContext *, int, int);

typedef struct {
    double t[6];          /* ligand translation and rotation */
    int    sc[40];        /* ligand sidechain rotations      */
} ligand;

int main(int argc, char **argv) {
    PGAContext *ctx;
    int maxiter;
    ctx = PGACreate(&argc, argv, PGA_DATATYPE_USER, 46, PGA_MINIMIZE);
    PGASetRandomSeed    (ctx, 1);
    PGASetMaxGAIterValue(ctx, 5000);
    PGASetUserFunction  (ctx, PGA_USERFUNCTION_CREATESTRING,  CreateString);
    PGASetUserFunction  (ctx, PGA_USERFUNCTION_MUTATION,      Mutation);
    PGASetUserFunction  (ctx, PGA_USERFUNCTION_CROSSOVER,     Crossover);
    PGASetUserFunction  (ctx, PGA_USERFUNCTION_PRINTSTRING,   WriteString);
    PGASetUserFunction  (ctx, PGA_USERFUNCTION_COPYSTRING,    CopyString);
    PGASetUserFunction  (ctx, PGA_USERFUNCTION_DUPLICATE,     DuplicateString);
    PGASetUserFunction  (ctx, PGA_USERFUNCTION_BUILDDATATYPE, BuildDT);
    PGASetUp            (ctx);
    PGARun              (ctx, Evaluate);
    PGADestroy          (ctx);
    return (0);
}
```

Figure 8.1: Main Program for Structure Data Type

```
void CreateString(PGAContext *ctx, int p, int pop, int InitFlag) {
    int i;
    ligand *ligand_ptr;
    PGAIndividual *new;

    new = PGAGetIndividual(ctx, p, pop);
    if (!(new->chrom = malloc(sizeof(ligand)))) {
        fprintf(stderr, "No room for new->chrom");
        exit(1);
    }
    ligand_ptr = (ligand *)new->chrom;
    if (InitFlag) {
        for (i = 0; i < 3; i++)
            ligand_ptr->t[i] = PGARandom01(ctx, 0) * 20.0 - 10.0;
        for (i = 3; i < 6; i++)
            ligand_ptr->t[i] = PGARandom01(ctx, 0) * 6.28 - 3.14;
        for (i = 0; i < 40; i++)
            ligand_ptr->sc[i] = PGARandomInterval(ctx, -20, 20);
    } else {
        for (i = 0; i < 6; i++)
            ligand_ptr->t[i] = 0.0;
        for (i = 0; i < 40; i++)
            ligand_ptr->sc[i] = 0;
    }
}
```

Figure 8.2: Creation and Initialization Function for Structure Data Type

ligand molecule about each of the axes. The `int`s in the `sc` array represent side chain rotations (which are discrete) of the ligand molecule.

Figure 8.2 contains the function `CreateString` that allocates and initializes the ligand structure. At this level of usage it is no longer always possible to maintain the `(p,pop)` abstraction to specify an individual (a string and associated fields). `CreateString` works directly with the string pointer that `(p,pop)` is mapped to. If `InitFlag` is true, `CreateString` will initialize the fields; otherwise they are set to 0.

`PGAGetIndividual(ctx, p, pop)` returns a pointer of type `PGAIndividual` to the individual (the string and associated fields) specified by `(p,pop)`. `PGAIndividual` is a structure, one of the fields of which is `chrom`, a `void` pointer to the string itself. That pointer, `new->chrom`, is assigned the address of the memory allocated by the `malloc` function. As `malloc` returns a `void` pointer, no cast is necessary.

The value of `InitFlag` is passed by PGAPack to the user's string creation routine. It specifies whether to randomly initialize the string or set it to zero. By default, `PGA_OLDPOP` (except for `PGA_TEMP1` and `PGA_TEMP1` which are set to zero) is randomly initialized, and `PGA_NEWPOP` is set to zero. This choice may be changed with the `PGASetRandomInitFlag` function discussed in Section 5.4.)

Figure 8.3 contains the mutation function `Mutation` for the ligand data type. Each of the 46 genes has a probability of `mr` of being changed. If a mutation occurs, `Mutation` adds or subtracts one tenth to the existing value of a `double`, and adds or subtracts one to an `int`.

Figure 8.4 contains the crossover function `Crossover`, which implements uniform crossover for the ligand data type. The lines

```
    parent1 = (ligand *)PGAGetIndividual(ctx, p1, pop1)->chrom;
    parent2 = (ligand *)PGAGetIndividual(ctx, p2, pop1)->chrom;
    child1  = (ligand *)PGAGetIndividual(ctx, t1, pop2)->chrom;
    child2  = (ligand *)PGAGetIndividual(ctx, t2, pop2)->chrom;
```

```
int Mutation(PGAContext *ctx, int p, int pop, double mr) {
    ligand *ligand_ptr;
    int i, count = 0;

    ligand_ptr = (ligand *)PGAGetIndividual(ctx, p, pop)->chrom;
    for (i = 0; i < 6; i++)
        if (PGARandomFlip(ctx, mr)) {
            if (PGARandomFlip(ctx, 0.5))
                ligand_ptr->t[i] += 0.1*ligand_ptr->t[i];
            else
                ligand_ptr->t[i] -= 0.1*ligand_ptr->t[i];
            count++;
        }
    for (i = 0; i < 40; i++)
        if (PGARandomFlip(ctx, mr)) {
            if (PGARandomFlip(ctx, 0.5))
                ligand_ptr->sc[i] += 1;
            else
                ligand_ptr->sc[i] -= 1;
            count++;
        }
    return (count);
}
```

Figure 8.3: Mutation for Structure Data Type

are worthy of mention. Each implements in one line what the two lines

```
new = PGAGetIndividual(ctx, p, pop);
string = (ligand *)new->chrom;
```

in `Mutation` did. Either style is acceptable. `PGAGetIndividual` returns a pointer whose `chrom` field (a `void` pointer) is cast to the `ligand` data type.

Figure 8.5 contains the code for `DuplicateString`, which checks for duplicate ligand structures. It uses the ANSI C `memcmp` function for this purpose.

Figure 8.6 contains the evaluation function for this example. It again uses `PGAGetIndividual` to map `(p, pop)` into a pointer to the string of interest. For user data types, PGAPack does *not* provide a `PGAGetUserAllele` function, so access to the allele values is made directly through the pointer.

Figure 8.7 contains the function `BuildDT` that builds an MPI datatype for sending strings to other processors. Consult an MPI manual for further information.

```
void Crossover(PGAContext *ctx, int p1, int p2, int pop1, int t1, int t2,
               int pop2) {
    int i;
    ligand *parent1, *parent2, *child1, *child2;
    double pu;

    parent1 = (ligand *)PGAGetIndividual(ctx, p1, pop1)->chrom;
    parent2 = (ligand *)PGAGetIndividual(ctx, p2, pop1)->chrom;
    child1  = (ligand *)PGAGetIndividual(ctx, t1, pop2)->chrom;
    child2  = (ligand *)PGAGetIndividual(ctx, t2, pop2)->chrom;

    pu = PGAGetUniformCrossoverProb(ctx);

    for (i = 0; i < 6; i++)
            if (PGARandomFlip(ctx, pu)) {
                child1->t[i] = parent1->t[i];
                child2->t[i] = parent2->t[i];
            } else {
                child1->t[i] = parent2->t[i];
                child2->t[i] = parent1->t[i];
            }
    for (i = 0; i < 40; i++)
            if (PGARandomFlip(ctx, pu)) {
                child1->sc[i] = parent1->sc[i];
                child2->sc[i] = parent2->sc[i];
            } else {
                child1->sc[i] = parent2->sc[i];
                child2->sc[i] = parent1->sc[i];
            }
}
```

Figure 8.4: Crossover for Structure Data Type

```
int DuplicateString(PGAContext *ctx, int p1, int pop1, int p2, int pop2) {
    void *a, *b;

     a = PGAGetIndividual(ctx, p1, pop1)->chrom;
     b = PGAGetIndividual(ctx, p2, pop2)->chrom;
     return (!memcmp(a, b, sizeof(ligand)));
}
```

Figure 8.5: Duplicate Testing for Structure Data Type

```
double Evaluate(PGAContext *ctx, int p, int pop) {
    int i, j;
    double x[6];
    int sc[40];
    PGAIndividual *ind;
    ligand *lig;

    lig = (ligand *)PGAGetIndividual(ctx, p, pop)->chrom;
    for (i = 0; i < 6; i++)
        x[i] = lig->t[i];
    for (i = 0; i < 40; i++)
        sc[i] = lig->sc[i];
    return ( energy(x,sc) );
}
```

Figure 8.6: Evaluation Function for Structure Data Type

```
MPI_Datatype BuildDT(PGAContext *ctx, int p, int pop) {
  int             counts[5];
  MPI_Aint        displs[5];
  MPI_Datatype    types[5];
  MPI_Datatype    DT_PGAIndividual;
  PGAIndividual  *P;
  ligand         *S;

  P = PGAGetIndividual(ctx, p, pop);
  S = (ligand *)P->chrom;

  /*  Build the MPI datatype.  Every user defined function needs these.
   *  The first two calls are stuff that is internal to PGAPack, but
   *  the user still must include it.  See pgapack.h for details one the
   *  fields (under PGAIndividual)
   */
  MPI_Address(&P->evalfunc, &displs[0]);
  counts[0] = 2;
  types[0]  = MPI_DOUBLE;

  /*  Next, we have an integer, evaluptodate.  */
  MPI_Address(&P->evaluptodate, &displs[1]);
  counts[1] = 1;
  types[1]  = MPI_INT;

  /*  Finally, we have the actual user-defined string.  */
  MPI_Address(S->t, &displs[2]);
  counts[2] = 6;
  types[2]  = MPI_DOUBLE;

  MPI_Address(S->sc, &displs[3]);
  counts[3] = 40;
  types[3]  = MPI_INT;

  MPI_Type_struct(4, counts, displs, types, &DT_PGAIndividual);
  MPI_Type_commit(&DT_PGAIndividual);
  return(DT_PGAIndividual);
}
```

Figure 8.7: Message Packing Function for Structure Data Type

Chapter 9

Hill-Climbing and Hybridization

Hill-climbing heuristics attempt to improve a solution by moving to a better *neighbor* solution. Whenever the neighboring solution is better than the current solution, it replaces the current solution. Genetic algorithms and hill-climbing heuristics have complementary strong and weak points. GAs are good at finding promising areas of the search space, but not as good at fine-tuning within those areas. Hill-climbing heuristics, on the other hand, are good at fine-tuning, but lack a global perspective. Practice has shown that a *hybrid* algorithm that combines GAs with hill-climbing heuristics often results in an algorithm that can outperform either one individually.

There are two general schemes for creating hybrid algorithms. The simplest is to run the genetic algorithm until it terminates and then apply a hill-climbing heuristic to each (or just the best) string. The second approach is to integrate a hill-climbing heuristic with the genetic algorithm. Choices to be made in the second case include how often to apply the hill-climbing heuristic and how many strings in the population to apply it to.

PGAPack supports hybrid schemes in the following ways:

- By passing, the context variable as a parameter to the user's hill-climbing function, the user has access to solution and parameter values, debug flags, and other information.
- The functions `PGAGetBinaryAllele`, `PGAGetIntegerAllele`, `PGAGetRealAllele`, and `PGAGetCharacterAllele` allow the user's hill-climbing function to read allele values, and the functions `PGASetBinaryAllele`, `PGASetIntegerAllele`, `PGASetRealAllele`, and `PGASetCharacterAllele` allow the user's hill-climbing function to set allele values explicitly.
- The functions `PGADecodeRealAsBinary`, `PGADecodeRealAsGrayCode`, `PGADecodeIntegerAsBinary`, and `PGADecodeIntegerAsGrayCode` allow the user's hill-climbing function to read integer or real numbers encoded as binary or Gray code strings.
- The functions `PGAEncodeRealAsBinary`, `PGAEncodeRealAsGrayCode`, `PGAEncodeIntegerAsBinary`, and `PGAEncodeIntegerAsGrayCode` allow the user's hill-climbing function to encode integer or real numbers as binary or Gray code strings.
- The functions `PGAGetEvaluation` and `PGASetEvaluation` allow the user's hill-climbing function to get and set evaluation function values, and `PGASetEvaluationUpToDateFlag` and `PGAGetEvaluationUpToDateFlag` to get and set the flag that indicates whether an evaluation function value is up to date.

One way to run a hybrid GA and use `PGARun` is to use the `PGASetUserFunction` discussed in Chapter 7 to specify a user function to be called at the end of each GA iteration. A more flexible approach would be for the user to call the high-level PGAPack functions, and their hillclimber to explicitly specify the steps of the hybrid GA.

Figure 9.1 is a version of the Maxbit problem given in Section 3.1. It uses the hill-climbing function `hillclimb`, which is called after the recombination step. It randomly selects a gene to set to one. Note the `PGASetEvaluationUpToDateFlag` call. It sets the flag that indicates the evaluation function is not current with the string (since the string was changed). It is *critical* that this flag be set when the user changes a

string, since the value of this flag determines whether `PGAEvaluate` will invoke the user's function evaluation routine.

```
#include "pgapack.h"

double evaluate(PGAContext *, int, int);
void hillclimb (PGAContext *, int);

int main(int argc, char **argv)
{
     PGAContext *ctx;

     ctx = PGACreate(&argc, argv, PGA_DATATYPE_BINARY, 100, PGA_MAXIMIZE);
     PGASetUp   (ctx);
     PGAEvaluate(ctx, PGA_OLDPOP, evaluate, NULL);
     PGAFitness (ctx, PGA_OLDPOP);
     while(!PGADone(ctx, NULL)) {
         PGASelect                 (ctx, PGA_OLDPOP);
         PGARunMutationAndCrossover(ctx, PGA_OLDPOP, PGA_NEWPOP);
         hillclimb                 (ctx, PGA_NEWPOP);
         PGAEvaluate               (ctx, PGA_NEWPOP, evaluate, NULL);
         PGAFitness                (ctx, PGA_NEWPOP);
         PGAUpdateGeneration       (ctx, NULL);
         PGAPrintReport            (ctx, stdout, PGA_OLDPOP);
     }
     PGADestroy(ctx);
     return 0;
}

void hillclimb(PGAContext *ctx, int pop)
{
     int i, p, popsize, stringlen;
     popsize   = PGAGetPopSize(ctx);
     stringlen = PGAGetStringLength(ctx);
     for (p=0; p<popsize; p++) {
         i = PGARandomInterval(ctx, 0, stringlen-1);
         PGASetBinaryAllele   (ctx, p, pop, i, 1);
         PGASetEvaluationUpToDateFlag   (ctx, p, pop, PGA_FALSE);
     }
}
```

Figure 9.1: Hill-Climbing Heuristic for Maxbit Example

Chapter 10

Parallel Aspects

This chapter assumes familiarity with the background material in Appendix C. It also assumes that a parallel version of PGAPack was built and that programs are linked with an MPI library (see Section 2.4).

Version 1.0 of PGAPack supports parallel and sequential *implementations* of the single population global model (GM). The parallel implementation uses a master/slave algorithm in which one process, the *master*, executes all steps of the genetic algorithm *except* the function evaluations. The function evaluations are executed by the *slave* processes[1].

10.1 Basic Usage

Both sequential and parallel versions of PGAPack may be run by using `PGARun`. The choice of sequential or parallel execution depends on the number of processes specified when the program is started. If one process is specified, the sequential implementation of the GM is used (even in a parallel version of PGAPack). If two or more processes are specified, the parallel implementation of the GM is used. The examples in Chapter 3 can all be run in parallel by specifying more than one process at startup.

The specification of the number of processors is done at run time. The actual format of the specification depends on the MPI implementation and computer used (see Appendix C for some examples). `PGARun` uses the default MPI communicator, `MPI_COMM_WORLD`. This specifies that all processes specified at startup participate in the computation: one as the master process, the others as slave processes. A different communicator may be specified with `PGASetCommunicator(ctx, comm)`, where `comm` is an MPI communicator.

`PGARun` is really a "wrapper" function that calls `PGARunGM` with the `MPI_COMM_WORLD` communicator. The user may call `PGARunGM` directly, that is, `PGARunGM(ctx,evaluate,MPI_COMM_WORLD)` where `evaluate` is the name of the user's evaluation function and the third argument is an MPI communicator. Note that the communicator specified by `PGASetCommunicator` does *not* affect `PGARunGM`.

10.2 Explicit Use

In general, explicit use of the parallel features is more complicated than in the case of sequential functions. This is because the user's program must coordinate the execution threads of *multiple* processes. `PGARunGM` encapsulates all that is necessary into one routine, and parts of its source code may serve as a useful starting point if one wishes to develop an explicitly parallel program. The parallel functions in PGAPack may be viewed as a hierarchy with `PGARun` and `PGARunGM` at the top of the hierarchy, `PGAEvaluate` next, `PGASendIndividual`, `PGAReceiveIndividual`, and `PGASendReceiveIndividual` next, and `PGABuildDatatype` at the bottom of the hierarchy.

`PGAGetRank(ctx,comm)` returns the rank of the process in communicator `comm`. If `comm` is `NULL` it returns 0. `PGAGetNumProcs(ctx,comm)` returns the number of processes in communicator `comm`. If `comm` is `NULL` it returns 1.

[1] In the special case of exactly two processes, the master executes function evaluations as well.

The type of algorithm used to execute `PGAEvaluate(ctx,pop,f,comm)` will depend on the number of processes in the communicator `comm`. If it is `NULL` or contains one process, a sequential implementation will be used. If more than one process is specified it will execute a master/slave evaluation of the strings in population `pop` that require evaluation by applying, `f`, the user's evaluation function. `PGAEvaluate` should be called by *all* processes in communicator `comm`.

`PGASendIndividual(ctx,p,pop,dest,tag,comm)` will send string `p` in population `pop` to process `dest`. `tag` is a tag used to identify the message, and `comm` is an MPI communicator. This function calls `MPI_Send` to perform the actual message passing. In addition to string `p` itself, the evaluation function value, fitness function value, and evaluation status flag are also sent.

`PGAReceiveIndividual` is the complementary function to `PGASendIndividual`. For example, `PGAReceiveIndividual (ctx,p,pop,source,tag,comm,status)` will store in location `p` in population `pop` the string and fields of the individual sent from process `source` with the MPI tag `tag` and MPI communicator `comm`. `status` is an MPI status vector.

`PGASendReceiveIndividual` combines the functionality of `PGASendIndividual` and `PGAReceiveIndividual`. This may be useful in avoiding potential deadlock on some systems. For example, `PGASendReceiveIndividual (ctx,sp,spop,dest,stag,rp,rpop,source, rtag,comm,status)`. Here, `sp` is the index of the string in population `spop` to send to process `dest` with tag `stag`. The string received from process `source` with tag `rtag` is stored in location `rp` in population `rpop`. `comm` and `status` are the same as defined earlier.

`PGABuildDatatype(ctx,p,pop)` packs together the string and fields that `PGASendIndividual`, `PGAReceiveIndividual`, and `PGASendReceiveIndividual` send and receive. The result is of type `MPI_Datatype`.

10.3 Example

Figure 10.1 is a parallel version of the example in Figure 6.1. Since we now have *multiple* processes executing the program at the same time, we must coordinate each ones execution. In the example, the master process (the one with rank 0 as determined by `PGAGetRank`) executes all functions, and the slave processes execute only those functions that take a communicator as an argument. Note that this example will execute correctly even if only one process is in the communicator.

10.4 Performance

The parallel implementation of the GM will produce the *same* result as the sequential implementation, usually faster. However, the parallel implementation varies with the number of processes. If two processes are used, both the master process and the slave process will compute the function evaluations. If more than two processes are used, the master is responsible for bookkeeping only, and the slaves for executing the function evaluations. In general, the speedup obtained will vary with the amount of computation associated with a function evaluation and the computational overhead of distributing and collecting information to and from the slave processes.

The speedup that can be achieved with the master/slave model is limited by the number of function evaluations that can be executed in parallel. This number depends on the population size and the number of new strings created each generation. For example, if the population size is 100 and a 100 new strings are created each GA generation, then up to 100 processors can be put to effective use to run the slave processes. However, with the default rule of replacing only 10% of the population each GA generation, only 10 processors can be used effectively.

```
#include "pgapack.h"
double evaluate (PGAContext *ctx, int p, int pop);

int main(int argc, char **argv)
{
    PGAContext *ctx;
    int rank;

    ctx = PGACreate(&argc, argv, PGA_DATATYPE_BINARY, 100, PGA_MAXIMIZE);
    PGASetUp   (ctx);
    rank = PGAGetRank(ctx, MPI_COMM_WORLD);
    PGAEvaluate(ctx, PGA_OLDPOP, evaluate, MPI_COMM_WORLD);
    if ( rank == 0 )
        PGAFitness (ctx, PGA_OLDPOP);
    while(!PGADone(ctx, MPI_COMM_WORLD)) {
        if ( rank == 0 ) {
            PGASelect                 (ctx, PGA_OLDPOP);
            PGARunMutationAndCrossover(ctx, PGA_OLDPOP, PGA_NEWPOP);
        }
        PGAEvaluate(ctx, PGA_OLDPOP, evaluate, MPI_COMM_WORLD);
        if ( rank == 0 )
            PGAFitness                (ctx, PGA_NEWPOP);
        PGAUpdateGeneration           (ctx, MPI_COMM_WORLD);
        if ( rank == 0 )
            PGAPrintReport            (ctx, stdout, PGA_OLDPOP);
    }
    PGADestroy(ctx);
    return(0);
}
```

Figure 10.1: Simple Parallel Example of Explicit Usage

Chapter 11

Fortran Interface

PGAPack is written entirely in ANSI C. A set of interface functions, also written in C, is designed to be called by Fortran programs and then call the "real" C routine. This mechanism provides most of PGAPack's functionality to Fortran programs. The following list contains most major differences between C and Fortran. Additional, machine-specific idiosyncrasies are noted in Appendix D.

- The `Makefile`s for the Fortran examples (in `./examples/fortran` and `./examples/mgh`) are *not* configured to use the -I mechanism for specifying the include file search path (since not all Fortran compilers support this). Therefore, you will need to copy or set up a symbolic link to `./include/pgapackf.h` from the directory you are compiling a Fortran program in.
- The context variable is declared `integer` (or `integer*8`, see Appendix D) in Fortran.
- `PGACreate` takes only three arguments in Fortran (not `argc` or `argv` as in C).
- The Fortran include file is `pgapackf.h` and should be included in any Fortran subroutine or function that calls a PGAPack function, to ensure correct typing and definition of functions and symbolic constants.
- If a C function returns an { `int`, `double`, pointer}, the corresponding Fortran function returns an { `integer`, `double precision`, `integer`}. If the C function is `void` it is implemented as a Fortran subroutine.
- When supplying function arguments, a C `int` corresponds to a Fortran `integer`, and a C `double` corresponds to a Fortran `double precision`. For example, to set the crossover probability to 0.6, use
  `call PGASetCrossoverProb(ctx, 0.6d0)`,
  or
  ```
  double precision pc
  pc = 0.6
  call PGASetCrossoverProb(ctx, pc)
  ```
- Gene indices are $[0, L-1]$ in C, and $[1, L]$ in Fortran, where L is the string length.
- Population member indices are $[0, N-1]$ in C, and $[1, N]$ in Fortran, where N is the population size.
- Fortran does not support command line arguments (Section 5.13).
- Fortran allows custom usage with native data types (Chapter 7), but not with new data types (Chapter 8).
- In the `MPICH` implementation of MPI, the Fortran and C versions of `MPI_Init` are different. If the main program is in C, then the C version of `MPI_Init` must be called. If the main program is in Fortran, the Fortran version of `MPI_Init` must be called. Therefore, Fortran users of PGAPack with `MPICH` must call `MPI_Init` themselves since `PGACreate`, which calls `MPI_Init` if users haven't called it themselves, is written in C.

- The DEC Alpha and Silicon Graphics Power Challenge, which have 64-bit C pointers and 32-bit Fortran integers (but not the Cray T3D which has 64-bit Fortran integers), have additional differences[1]. These arise because a Fortran integer is too small to hold the address returned by the C interface routine.
  - The context variable should be declared `integer*8`.
  - `MPI_COMM_WORLD` should *not* be passed directly to PGAPack Fortran functions. Instead, `PGAGetCommunicator` should be called to return the address into an `integer*8` variable. For example

    ```
    integer pid
    integer*8 comm
    comm = PGAGetCommunicator(ctx)
    :
    pid  = PGAGetRank(ctx, comm)
    ```

  - `MPI_COMM_WORLD` *can* and *should* be passed directly to any MPI routines called directly from Fortran.
  - Calling an MPI routine that returns a communicator is safe. However, passing the returned communicator to a PGAPack Fortran function will usually fail.

[1] More generally, these issues arise whenever the size of a Fortran integer is less than the size of a pointer.

Chapter 12

Debugging Tools

PGAPack has a sophisticated built-in trace facility that is useful for debugging. When the facility is invoked, print statements to `stdout` allow the programmer to trace the sequence of functions PGAPack executes. Due to the negative impact on performance this facility is *not* available by default. Instead, you must explicitly enable tracing when configuring PGAPack with the `-debug` flag. See Section 2.4.

The trace facility uses the concept of a *debug level*. For example, executing the Maxbit example (Figure 3.1) with a debug level of 12, `maxbit -pgadbg 12`, will print the output shown in Figure 12.1. The "`0:`" is the process rank. This is followed by the name of a PGAPack function and the "action" that caused the print statement to execute. In this case, the action is entering the function. Note that the rank printed for a process is *always* its rank in the `MPI_COMM_WORLD` communicator, even if another communicator was set.

Tracing is enabled by specifying one or more debug levels to trace. A list of debug levels is given in Table 12.1. Not all debug level values are currently used. The values 1–10 are reserved for users as described below.

C programmers may set the debug level from the command line using either `-pgadbg <debug_level>` or `-pgadebug <debug_level>`. Several forms of the `<debug_level>` argument are allowed. `-pgadbg 12` will trace entering all high-level functions as shown in Figure 12.1. `-pgadbg 12,13` or `-pgadbg 12-13` will trace entering *and* exiting of all high-level functions. The command line option `-pgahelp debug` will list the debug level options and then exit.

Fortran (and C) programmers may access the trace facility via function calls. The function `PGASetDebugLevel` may be called to set a debug level. For example, `call PGASetDebugLevel(ctx,12)` would produce the same output shown in Figure 12.1. `PGAClearDebugLevel(ctx,12)` will clear prints associated with debug level 12. `PGAPrintDebugOptions(ctx)` will print the list of available debug options.

The function `PGASetDebugLevelByName` will turn on debugging of the named function. For example, `PGASetDebugLevelByName(ctx,''PGACrossover'')` will enable all the trace prints of `PGACrossover`. `PGAClearDebugLevelByName` will disable the tracing of the specified function.

Users can use the trace facility in their own functions (e.g., their evaluation function) in two ways. First, they can insert `PGADebugPrint` function calls in their functions using one of the symbolic constants defined in the header file `pgapack.h`. These are `PGA_DEBUG_ENTERED`, `PGA_DEBUG_EXIT`, `PGA_DEBUG_MALLOC`, `PGA_DEBUG_PRINTVAR`, `PGA_DEBUG_SEND`, and `PGA_DEBUG_RECV` for entering a function, exiting a function, allocating memory, print a variable's value, and sending or receiving a string, respectively.

For example, `PGADebugPrint(ctx, PGA_DEBUG_ENTERED, "MyFunc", "Entered", PGA_VOID, NULL)` will print the line

```
0: MyFunc                       : Entered
```

when the debug level of 12 is specified. `PGADebugPrint(ctx, PGA_DEBUG_PRINTVAR, "MyFunc", "i = ", PGA_INT, (void *) &i)` will print the line

```
0: MyFunc                       : i =  1
```

when the debug level of 82 is specified. Users can also use the reserved debug levels of 1–10 to customize the trace facilities for use in their own functions. For example `PGADebugPrint( ctx, 5, "MyFunc", "After call to MyCleanUp", PGA_VOID, NULL)`; will print the line

```
0: PGACreate                        : Entered
0: PGASetRandomSeed                 : Entered
0: PGASetMaxGAIterValue             : Entered
0: PGASetUp                         : Entered
0: PGACreatePop                     : Entered
0: PGACreateIndividual              : Entered
:
:
0: PGACreateIndividual              : Entered
0: PGACreatePop                     : Entered
0: PGACreateIndividual              : Entered
:
:
0: PGARun                           : Entered
0: PGARunSeq                        : Entered
0: PGAEvaluate                      : Entered
0: PGAFitness                       : Entered
0: PGAGetStringLength               : Entered
:
:
```

Figure 12.1: PGAPack Partial Trace Output for Maxbit Example

```
0: MyFunc                           : After call to MyCleanUp
```

when the debug level of five is specified.

Note that we use `MPI_COMM_WORLD` (1) for the random number seed and (2) for `PGADebugPrint` calls.

Table 12.1: Debug Levels in PGAPack

0	Trace all debug prints
11	Trace high-level functions
12	Trace all function entries
13	Trace all function exits
20	Trace high-level parallel functions
21	Trace all parallel functions
22	Trace all send calls
23	Trace all receive calls
30	Trace Binary functions
32	Trace Integer functions
34	Trace Real functions
36	Trace Character functions
40	Trace population creation functions
42	Trace select functions
44	Trace mutation functions
46	Trace crossover functions
48	Trace function evaluation functions
50	Trace fitness calculation functions
52	Trace duplicate checking functions
54	Trace restart functions
56	Trace reporting functions
58	Trace stopping functions
60	Trace sorting functions
62	Trace random number functions
64	Trace system routines
66	Trace utility functions
80	Trace memory allocations
82	Trace variable print statements

Part III

Appendixes

Appendix A

Default Values

Table A.1: PGAPack Default Values

CONCEPT	DEFAULT	SET WITH
Population size	100	PGASetPopSize
Copied for population replacement	PGA_POPREPL_BEST	PGASetPopReplacementType
Stopping rule	PGA_STOP_MAXITER	PGASetStoppingRuleType
Maximum iterations	1000	PGASetMaxGAIterValue
Maximum no change iters	100	PGASetMaxNoChangeValue
Max. population homogeneity before stopping	95	PGASetMaxSimilarityValue
Number of new strings to generate	10	PGASetNumReplaceValue
Apply mutation and crossover	PGA_FALSE	PGASetMutationAndCrossoverFlag
Apply mutation or crossover	PGA_TRUE	PGASetMutationOrCrossoverFlag
Crossover type	PGA_CROSSOVER_TWOPT	PGASetCrossoverType
Probability of crossover	0.85	PGASetCrossoverProb
Uniform crossover bias	0.6	PGASetUniformCrossoverProb
Mutation type (Real strings)	PGA_MUTATION_GAUSSIAN	PGASetMutationType
Mutation type (Integer strings)	PGA_MUTATION_PERMUTE	PGASetMutationType
Mutation type (Character strings)	Same as initialization	PGASetCharacterInitType
Mutation probability	1/L	PGASetMutationProb
Real mutation constant	0.1	PGASetMutationRealValue
Integer mutation constant	1	PGASetMutationIntegerValue
Mutation range bounded	PGA_TRUE	PGASetMutationBoundedFlag
Select type	PGA_SELECT_TOURNAMENT	PGASetSelectType
Probabilistic binary tournament parameter	0.6	PGASetPTournamentProb
Use restart operator	PGA_FALSE	PGASetRestartFlag
Restart frequency	50	PGASetRestartFrequencyValue
Restart allele mutation rate	0.5	PGASetRestartAlleleChangeProb
Allow duplicate strings	PGA_FALSE	PGASetNoDuplicatesFlag
Fitness type	PGA_FITNESS_RAW	PGASetFitnessType
Fitness type for minimization	PGA_FITNESSMIN_CMAX	PGASetFitnessMinType
Multiplier for minimization problems	1.01	PGASetCMaxValue
Parameter MAX in fitness by ranking	1.2	PGASetMaxFitnessRank
Frequency of statistics printing	10	PGASetPrintFrequencyValue
Print strings	PGA_FALSE	PGASetPrintOptions
Print offline statistics	PGA_FALSE	PGASetPrintOptions
Print online statistics	PGA_FALSE	PGASetPrintOptions
Print best string	PGA_FALSE	PGASetPrintOptions
Print worst string	PGA_FALSE	PGASetPrintOptions
Print Hamming distance	PGA_FALSE	PGASetPrintOptions
Randomly initialize population	PGA_TRUE	PGASetRandomInitFlag
Probability of initializing a bit to one	0.5	PGASetBinaryInitProb
How to initialize real strings	Range	PGASetrealInitRange
Real initialization range	$[0, 1]$	PGASetRealInitRange
How to initialize integer strings	Permutation	PGASetIntegerInitPermute
Integer initialization range	$[0, L-1]$	PGASetIntegerInitPermute
How to initialize character strings	PGA_CINIT_LOWER	PGASetCharacterInitFlag
Seed random number with clock	PGA_TRUE	PGASetRandomSeed
Default MPI communicator	MPI_COMM_WORLD	PGASetCommunicator

L is the string length

Appendix B

Function Bindings

Symbolic Constants

PGAPack defines many symbolic constants that are used as arguments to PGAPack functions. These constants are the same for both Fortran and C. Below is a list of these constants. These constants are the same for both Fortran and C.

- PGAPack Data Types
  - `PGA_DATATYPE_BINARY`
  - `PGA_DATATYPE_INTEGER`
  - `PGA_DATATYPE_REAL`
  - `PGA_DATATYPE_CHARACTER`
  - `PGA_DATATYPE_USER`
- String Types
  - `PGABinary`
  - `PGAInteger`
  - `PGAReal`
  - `PGACharacter`
- Data Types used in `PGAError` Calls
  - `PGA_INT`
  - `PGA_DOUBLE`
  - `PGA_CHAR`
  - `PGA_VOID`
- True and False
  - `PGA_TRUE`
  - `PGA_FALSE`
- Miscellaneous PGAPack Flags
  - `PGA_FATAL`
  - `PGA_WARNING`
  - `PGA_UNINITIALIZED_INT`

  - PGA_UNINITIALIZED_DOUBLE

- PGAPack Temporary and Population Constants
  - PGA_TEMP1
  - PGA_TEMP2
  - PGA_OLDPOP
  - PGA_NEWPOP

- Debug Levels
  - PGA_DEBUG_ENTERED
  - PGA_DEBUG_EXIT
  - PGA_DEBUG_MALLOC
  - PGA_DEBUG_PRINTVAR
  - PGA_DEBUG_SEND
  - PGA_DEBUG_RECV

- Direction of Optimization
  - PGA_MAXIMIZE
  - PGA_MINIMIZE

- Stopping Criteria
  - PGA_STOP_MAXITER
  - PGA_STOP_NOCHANGE
  - PGA_STOP_TOOSIMILAR

- Crossover
  - PGA_CROSSOVER_ONEPT
  - PGA_CROSSOVER_TWOPT
  - PGA_CROSSOVER_UNIFORM

- Fitness
  - PGA_FITNESS_RAW
  - PGA_FITNESS_NORMAL
  - PGA_FITNESS_RANKING

- Fitness Minimization
  - PGA_FITNESSMIN_RECIPROCAL
  - PGA_FITNESSMIN_CMAX

- Mutation Type
  - PGA_MUTATION_CONSTANT
  - PGA_MUTATION_RANGE
  - PGA_MUTATION_UNIFORM
  - PGA_MUTATION_GAUSSIAN
  - PGA_MUTATION_PERMUTE

- Population Replacement
  - PGA_POPREPL_BEST
  - PGA_POPREPL_RANDOM_NOREP
  - PGA_POPREPL_RANDOM_REP
- Initialization Options
  - PGA_CINIT_LOWER
  - PGA_CINIT_UPPER
  - PGA_CINIT_MIXED
  - PGA_IINIT_PERMUTE
  - PGA_IINIT_RANGE
  - PGA_RINIT_PERCENT
  - PGA_RINIT_RANGE
- Report Options
  - PGA_REPORT_ONLINE
  - PGA_REPORT_OFFLINE
  - PGA_REPORT_HAMMING
  - PGA_REPORT_STRING
  - PGA_REPORT_WORST
  - PGA_REPORT_AVERAGE
- Selection
  - PGA_SELECT_PROPORTIONAL
  - PGA_SELECT_SUS
  - PGA_SELECT_TOURNAMENT
  - PGA_SELECT_PTOURNAMENT
- User Functions
  - PGA_USERFUNCTION_CREATESTRING
  - PGA_USERFUNCTION_MUTATION
  - PGA_USERFUNCTION_CROSSOVER
  - PGA_USERFUNCTION_PRINTSTRING
  - PGA_USERFUNCTION_COPYSTRING
  - PGA_USERFUNCTION_DUPLICATE
  - PGA_USERFUNCTION_INITSTRING
  - PGA_USERFUNCTION_BUILDDATATYPE
  - PGA_USERFUNCTION_STOPCOND
  - PGA_USERFUNCTION_ENDOFGEN

ANSI C Bindings

The use of any PGAPack function requires that the user have `#include "pgapack.h"` at the top of the file that references PGAPack functions.

Type	Function
MPI_Datatype	PGABuildDatatype(PGAContext *ctx, int p, int pop)
void	PGAChange(PGAContext *ctx, int p, int pop)
int	PGACheckStoppingConditions(PGAContext *ctx)
int	PGACheckSum(PGAContext *ctx, int p, int pop)
void	PGAClearDebugLevel(PGAContext *ctx, int level)
void	PGAClearDebugLevelByName(PGAContext *ctx, char *funcname)
void	PGACopyIndividual(PGAContext *ctx, int p1, int pop1, int p2, int pop2)
PGAContext*	PGACreate(int *argc, char **argv, int datatype, int len, int maxormin)
void	PGACrossover(PGAContext *ctx, int p1, int p2, int pop1, int c1, int c2, int pop2)
void	PGADebugPrint(PGAContext *ctx, int level, char *funcname, char *msg, int datatype, void *data)
void	PGADestroy(PGAContext *ctx)
int	PGADone(PGAContext *ctx, MPI_Comm comm)
int	PGADuplicate(PGAContext *ctx, int p, int pop1, int pop2, int n)
void	PGAEncodeIntegerAsBinary(PGAContext *ctx, int p, int pop, int start, int end, int val)
void	PGAEncodeIntegerAsGrayCode(PGAContext *ctx, int p, int pop, int start, int end, int val)
void	PGAEncodeRealAsBinary(PGAContext *ctx, int p, int pop, int start, int end, double low, double high, double val)
void	PGAEncodeRealAsGrayCode(PGAContext *ctx, int p, int pop, int start, int end, double low, double high, double val)
void	PGAError(PGAContext *ctx, char *msg, int level, int datatype, void *data)
void	PGAEvaluate(PGAContext *ctx, int pop, double(*f)(PGAContext *, int, int), MPI_Comm comm)
void	PGAFitness(PGAContext *ctx, int popindex)
int	PGAGetBestIndex(PGAContext *ctx, int pop)
int	PGAGetBinaryAllele(PGAContext *ctx, int p, int pop, int i)
double	PGAGetBinaryInitProb(PGAContext *ctx)
char	PGAGetCharacterAllele(PGAContext *ctx, int p, int pop, int i)
MPI_Comm	PGAGetCommunicator(PGAContext *ctx)
double	PGAGetCrossoverProb(PGAContext *ctx)
int	PGAGetCrossoverType(PGAContext *ctx)
int	PGAGetDataType(PGAContext *ctx)
double	PGAGetEvaluation(PGAContext *ctx, int p, int pop)
int	PGAGetEvaluationUpToDateFlag(PGAContext *ctx, int p, int pop)
double	PGAGetFitness(PGAContext *ctx, int p, int pop)
double	PGAGetFitnessCmaxValue(PGAContext *ctx)
int	PGAGetFitnessMinType(PGAContext *ctx)
int	PGAGetFitnessType(PGAContext *ctx)
int	PGAGetGAIterValue(PGAContext *ctx)
int	PGAGetIntegerAllele(PGAContext *ctx, int p, int pop, int i)
int	PGAGetIntegerFromBinary(PGAContext *ctx, int p, int pop, int start, int end)
int	PGAGetIntegerFromGrayCode(PGAContext *ctx, int p, int pop, int start, int end)
int	PGAGetIntegerInitType(PGAContext *ctx)

Type	Function
double	PGAGetMaxFitnessRank(PGAContext *ctx)
int	PGAGetMaxGAIterValue(PGAContext *ctx)
int	PGAGetMaxIntegerInitValue(PGAContext *ctx, int i)
double	PGAGetMaxMachineDoubleValue(PGAContext *ctx)
int	PGAGetMaxMachineIntValue(PGAContext *ctx)
double	PGAGetMaxRealInitValue(PGAContext *ctx, int i)
int	PGAGetMinIntegerInitValue(PGAContext *ctx, int i)
double	PGAGetMinMachineDoubleValue(PGAContext *ctx)
int	PGAGetMinMachineIntValue(PGAContext *ctx)
double	PGAGetMinRealInitValue(PGAContext *ctx, int i)
int	PGAGetMutationAndCrossoverFlag(PGAContext *ctx)
int	PGAGetMutationBoundedFlag(PGAContext *ctx)
int	PGAGetMutationIntegerValue(PGAContext *ctx)
int	PGAGetMutationOrCrossoverFlag(PGAContext *ctx)
double	PGAGetMutationProb(PGAContext *ctx)
double	PGAGetMutationRealValue(PGAContext *ctx)
int	PGAGetMutationType(PGAContext *ctx)
int	PGAGetNoDuplicatesFlag(PGAContext *ctx)
int	PGAGetNumProcs(PGAContext *ctx, MPI_Comm comm)
int	PGAGetNumReplaceValue(PGAContext *ctx)
int	PGAGetOptDirFlag(PGAContext *ctx)
double	PGAGetPTournamentProb(PGAContext *ctx)
int	PGAGetPopReplaceType(PGAContext *ctx)
int	PGAGetPopSize(PGAContext *ctx)
int	PGAGetPrintFrequencyValue(PGAContext *ctx)
int	PGAGetRandomInitFlag(PGAContext *ctx)
int	PGAGetRandomSeed(PGAContext *ctx)
int	PGAGetRank(PGAContext *ctx, MPI_Comm comm)
double	PGAGetRealAllele(PGAContext *ctx, int p, int pop, int i)
double	PGAGetRealFromBinary(PGAContext *ctx, int p, int pop, int start, int end, double lower, double upper)
double	PGAGetRealFromGrayCode(PGAContext *ctx, int p, int pop, int start, int end, double lower, double upper)
int	PGAGetRealInitType(PGAContext *ctx)
double	PGAGetRestartAlleleChangeProb(PGAContext *ctx)
int	PGAGetRestartFlag(PGAContext *ctx)
int	PGAGetRestartFrequencyValue(PGAContext *ctx)
int	PGAGetSelectType(PGAContext *ctx)
int	PGAGetSortedPopIndex(PGAContext *ctx, int n)
int	PGAGetStoppingRuleType(PGAContext *ctx)
int	PGAGetStringLength(PGAContext *ctx)
double	PGAGetUniformCrossoverProb(PGAContext *ctx)
int	PGAGetWorstIndex(PGAContext *ctx, int pop)
double	PGAHammingDistance(PGAContext *ctx, int popindex)
double	PGAMean(PGAContext *ctx, double *a, int n)
int	PGAMutate(PGAContext *ctx, int p, int pop)
void	PGAPrintContextVariable(PGAContext *ctx, FILE *fp)
void	PGAPrintIndividual(PGAContext *ctx, FILE *fp, int p, int pop)
void	PGAPrintPopulation(PGAContext *ctx, FILE *fp, int pop)
void	PGAPrintReport(PGAContext *ctx, FILE *fp, int pop)
void	PGAPrintString(PGAContext *ctx, FILE *file, int p, int pop)
void	PGAPrintVersionNumber(PGAContext *ctx)

Type	Function
double	PGARandom01(PGAContext *ctx, int newseed)
int	PGARandomFlip(PGAContext *ctx, double p)
double	PGARandomGaussian(PGAContext *ctx, double mean, double sigma)
int	PGARandomInterval(PGAContext *ctx, int start, int end)
double	PGARandomUniform(PGAContext *ctx, double start, double end)
int	PGARank(PGAContext *ctx, int p, int *order, int n)
void	PGAReceiveIndividual(PGAContext *ctx, int p, int pop, int source, int tag, MPI_Comm comm, MPI_Status *status)
void	PGARestart(PGAContext *ctx, int source_pop, int dest_pop)
int	PGARound(PGAContext *ctx, double x)
void	PGARun(PGAContext *ctx, double(*evaluate)(PGAContext *c, int p, int pop))
void	PGARunGM(PGAContext *ctx, double(*f)(PGAContext *, int, int), MPI_Comm comm)
void	PGARunMutationAndCrossover(PGAContext *ctx, int oldpop, int newpop)
void	PGARunMutationOrCrossover(PGAContext *ctx, int oldpop, int newpop)
void	PGASelect(PGAContext *ctx, int popix)
int	PGASelectNextIndex(PGAContext *ctx)
void	PGASendIndividual(PGAContext *ctx, int p, int pop, int dest, int tag, MPI_Comm comm)
void	PGASendReceiveIndividual(PGAContext *ctx, int send_p, int send_pop, int dest, int send_tag, int recv_p, int recv_pop, int source, int recv_tag, MPI_Comm comm, MPI_Status *status)
void	PGASetBinaryAllele(PGAContext *ctx, int p, int pop, int i, int val)
void	PGASetBinaryInitProb(PGAContext *ctx, double probability)
void	PGASetCharacterAllele(PGAContext *ctx, int p, int pop, int i, char value)
void	PGASetCharacterInitType(PGAContext *ctx, int value)
void	PGASetCommunicator(PGAContext *ctx, MPI_Comm comm)
void	PGASetCrossoverProb(PGAContext *ctx, double crossover_prob)
void	PGASetCrossoverType(PGAContext *ctx, int crossover_type)
void	PGASetDebugLevel(PGAContext *ctx, int level)
void	PGASetDebugLevelByName(PGAContext *ctx, char *funcname)
void	PGASetEvaluation(PGAContext *ctx, int p, int pop, double val)
void	PGASetEvaluationUpToDateFlag(PGAContext *ctx, int p, int pop, int status)
void	PGASetFitnessCmaxValue(PGAContext *ctx, double val)
void	PGASetFitnessMinType(PGAContext *ctx, int fitness_type)
void	PGASetFitnessType(PGAContext *ctx, int fitness_type)
void	PGASetIntegerAllele(PGAContext *ctx, int p, int pop, int i, int value)
void	PGASetIntegerInitPermute(PGAContext *ctx, int min, int max)
void	PGASetIntegerInitRange(PGAContext *ctx, int *min, int *max)
void	PGASetMaxFitnessRank(PGAContext *ctx, double fitness_rank_max)
void	PGASetMaxGAIterValue(PGAContext *ctx, int maxiter)
void	PGASetMaxNoChangeValue(PGAContext *ctx, int max_no_change)
void	PGASetMaxSimilarityValue(PGAContext *ctx, int max_similarity)
void	PGASetMutationAndCrossoverFlag(PGAContext *ctx, int flag)
void	PGASetMutationBoundedFlag(PGAContext *ctx, int val)
void	PGASetMutationIntegerValue(PGAContext *ctx, int val)
void	PGASetMutationOrCrossoverFlag(PGAContext *ctx, int flag)
void	PGASetMutationProb(PGAContext *ctx, double mutation_prob)
void	PGASetMutationRealValue(PGAContext *ctx, double val)
void	PGASetMutationType(PGAContext *ctx, int mutation_type)

Type	Function
void	PGASetNoDuplicatesFlag(PGAContext *ctx, int no_dup)
void	PGASetNumReplaceValue(PGAContext *ctx, int pop_replace)
void	PGASetPTournamentProb(PGAContext *ctx, double ptournament_prob)
void	PGASetPopReplaceType(PGAContext *ctx, int pop_replace)
void	PGASetPopSize(PGAContext *ctx, int popsize)
void	PGASetPrintFrequencyValue(PGAContext *ctx, int print_freq)
void	PGASetPrintOptions(PGAContext *ctx, int option)
void	PGASetRandomInitFlag(PGAContext *ctx, int RandomBoolean)
void	PGASetRandomSeed(PGAContext *ctx, int seed)
void	PGASetRealAllele(PGAContext *ctx, int p, int pop, int i, double value)
void	PGASetRealInitPercent(PGAContext *ctx, double *median, double *percent)
void	PGASetRealInitRange(PGAContext *ctx, double *min, double *max)
void	PGASetRestartAlleleChangeProb(PGAContext *ctx, double prob)
void	PGASetRestartFlag(PGAContext *ctx, int val)
void	PGASetRestartFrequencyValue(PGAContext *ctx, int numiter)
void	PGASetSelectType(PGAContext *ctx, int select_type)
void	PGASetStoppingRuleType(PGAContext *ctx, int stoprule)
void	PGASetUniformCrossoverProb(PGAContext *ctx, double uniform_cross_prob)
void	PGASetUp(PGAContext *ctx)
void	PGASetUserFunction(PGAContext *ctx, int constant, void *f)
void	PGASortPop(PGAContext *ctx, int pop)
double	PGAStddev(PGAContext *ctx, double *a, int n, double mean)
void	PGAUpdateGeneration(PGAContext *ctx, MPI_Comm comm)
void	PGAUsage(PGAContext *ctx)

Fortran 77 Bindings

Use the rules defined in Chapter 11 (and the machine-specific idiosyncrasies noted in Appendix D) to determine the Fortran bindings.

Appendix C

Parallelism Background

Parallel Computer Taxonomy

Traditionally, parallel computers are classified according to Flynn's taxonomy [4]. Flynn's classification distinguishes parallel computers according to the number of instruction streams and data operands being computed on simultaneously.

Flynn's single-instruction single-data (SISD) model is the traditional sequential computer. A single program counter fetches instructions from memory. The instructions are executed on *scalar* operands. There is no parallelism in this model.

In the single-instruction multiple-data (SIMD) model there is again a single program counter fetching instructions from memory. However, now the operands of the instructions can be one of two types: either scalar or array. If the instruction calls for execution involving only scalar operands, it is executed by the control processor (i.e., the central processing unit fetching instructions from memory). If, on the other hand, the instruction calls for execution using array operands, it is broadcast to the *array* of processing elements. The processing elements are separate computing devices that rely upon the control processor to determine the instructions they will execute.

In a multiple-instruction multiple-data (MIMD) computer there exist multiple processors each of which has its own program counter. Processors execute independently of each other according to whatever instruction the program counter points to next. MIMD computers are usually further subdivided according to whether the processors share memory or each has its own memory.

In a shared-memory MIMD computer both the program's instructions and the part of the program's data to be shared exist within a single shared memory. Additionally, some data may be private to a processor and not be globally accessible by other processors. The processors execute asynchronously of each other. Communication and synchronization between the processors are handled by having them each read or write a shared-memory location.

A distributed-memory MIMD computer consists of multiple "nodes." A node consists of a processor, its own memory, a network interface, and sometimes a local disk. The program instructions and data reside in the node's memory. The nodes are connected via some type of network that allows them to communicate with each other. Parallelism is achieved by having each processor compute simultaneously on the data in its own memory. Communication and synchronization are handled by passing of messages (a destination node address and the local data to be sent) over the interconnection network.

Processes vs. Processors

We distinguish the two terms process and processor. A *process* is a software abstraction with a unique address space that can be scheduled by the operating system. A *processor* is the physical computer hardware on which computations take place.

On MIMD parallel computers, usually one process executes on each processor (although this is not required). On a uniprocessor, multiple processes timeshare the single processor.

Message-Passing Programming Model

In the message-passing programming model multiple processes communicate by passing messages—transferring data from the address space of one process into the address space of another process. When a process needs data stored in the memory of another process, the data must be sent from the process that "owns" it, to the memory of the process that needs it.

The message-passing programming model is currently one of the most popular. One reason for the popularity is portability. Message passing is the natural programming model on distributed-memory MIMD computers. Each process is naturally mapped to one of the machine's nodes. A similar implementation is common on a workstation network where one process runs on each workstation. On a shared-memory MIMD computer multiple processes can emulate message passing by communicating only via logical message queues—areas of shared memory partitioned by process. On a uniprocessor the multiple processes that timeshare the physical processor can also emulate the idea of logical message queues for their communication.

One example of the message-passing programming model is the master/slave model. In this model a *master* process distributed work (computation to be performed) to the slave processes. The slaves perform the work and return the result to the master. In many implementations the master plays a bookkeeping role only and does not perform any computation.

Parallel Genetic Algorithms

When using the term "parallel genetic algorithm" it is important to distinguish between parallel models, their (parallel or sequential) implementation, and the computer hardware.

Models

A sequential GA model (more accurately called a *global* model) has a single population and no restrictions (partitioning) upon which strings recombine with which. The sequential GA is the traditional GA model given in the literature. In a parallel GA model there are either multiple populations (an island model) or a partitioning of a single population (often called a fine-grained model).

Implementations

Both parallel and sequential GA models can have parallel or sequential implementations. A sequential implementation of the global model is the traditional approach discussed in the GA literature. One process, running on a uniprocessor (PCs and workstations), performs all the calculations. In a parallel implementation of the global model the steps of the GA (some or all of selection, crossover, mutation, and fitness calculation) are executed simultaneously by multiple processes running on a parallel computer or workstation network.

In a sequential implementation of a parallel GA model, multiple processes, each responsible for a subpopulation or partition of the full population, time share the processor of the uniprocessor they are running on. In a parallel implementation of a parallel GA model, the multiple processes each run on a unique processor of a parallel computer or workstation network.

MPI

MPI (Message Passing Interface) is a *specification* of a message-passing library for parallel computers and workstation networks—it defines a set of functions and their behavior. The actual *implementation* of this interface is left up to vendors and researchers to develop. At the time of this writing several implementations of MPI, both proprietary and freely available, exist. MPI was designed by a large group of parallel computer vendors, computer researchers, and application developers as a standard for message passing.

Communicators

Almost all MPI functions require a *communicator*. If MPI routines are called directly, the user must supply a communicator. Also, if any of PGAPack's parallel routines, other than `PGARun`, are used, the user must supply a communicator as well.

A communicator combines the notions of context and group. A *context* is an extension of the notion of a "tag" used in many other message-passing systems to identify a message. Contexts differ from tags in that they are allocated by the system, not the user, and that no wild-card matching among contexts is allowed. A *group* contains n processes whose *rank* is an integer between $0, \ldots, n-1$. Processes may belong to more than one group and have a unique rank within each.

Any MPI implementation will always supply the default communicator `MPI_COMM_WORLD`. This communicator contains all processes that were created when MPI was initialized. For most users this is all they have to know about communicators. Simply supply `MPI_COMM_WORLD` whenever a communicator is required as an argument. For more sophisticated use, users are referred to [5, 6].

Parallel I/O

The issue of parallel I/O is independent of PGAPack. However, since we assume many PGAPack users will wish to do I/O we address this topic. A primary consideration has to do with whether one or all processors do I/O. Consider the following two code fragments, keeping in mind that they are being executed simultaneously by *multiple* processes:

```
ctx = PGACreate(&argc, argv, PGA_DATATYPE_BINARY, 30, PGA_MINIMIZE)
```

and

```
int len;
scanf("%d",&len);
ctx = PGACreate(&argc, argv, PGA_DATATYPE_BINARY, len, PGA_MINIMIZE);
```

In the first case, all processes will receive the value of 30 for the string length since it is a constant. In the second case, however, the value of the string length is determined at run time. Whether one or all processes execute the `scanf` function is undefined in MPI and depends on the particular parallel computing environment. In PGAPack we require that all processes have the same values for all fields in the context variable. Since some of these fields may be set by using values specified at run time, we suggest that your I/O that reads in PGAPack parameters be done as follows:

```
#include "pgapack.h"
double evaluate (PGAContext *ctx, int p, int pop);

int main( int argc, char **argv )
{
     PGAContext *ctx;
     int myid, len;

     MPI_Init(&argc, &argv);
     MPI_Comm_rank(MPI_COMM_WORLD, &myid);
     if (myid == 0) {                        /* Process 0 has a dialog */
         printf("String length? ");          /* with the user and      */
         scanf("%d", &len);                  /* broadcasts the user's  */
     }
     MPI_Bcast(&len, 1, MPI_INT, 0, MPI_COMM_WORLD);

     ctx = PGACreate(&argc, argv, PGA_DATATYPE_BINARY, len, PGA_MAXIMIZE);
     PGASetUp(ctx);
     PGARun(ctx, evaluate);
     PGADestroy(ctx);
```

```
    MPI_Finalize();
    return(0);
}
```

The key point is that *only* process 0 (as determined by `MPI_Comm_rank`) performs I/O and the value of `len` is then broadcast (using `MPI_Bcast`) to the other processes.

Appendix D

Machine Idiosyncrasies

Data Type Sizes

PGAPack is written entirely in ANSI C. However, because it is callable from Fortran, and no standards exist for interlanguage communication, problems may arise. These have to do with a lack of consistency in the size of data types between the two languages.

On all machines we have tested, an `integer` declaration in Fortran is the same size as an `int` declaration in C and everything works properly. For floating-point numbers, however, we have found at least one inconsistency. The requirement is for the Fortran floating-point number to be the same size as a C `double`. On most machines a Fortran `double precision` declaration is the equivalent size. On the Cray T3D, however, by default, the Fortran data type `double precision` is not supported and must be handled as described below.

Since Fortran does not support pointers, an `integer` variable is used to hold the address of the context variable (and possibly MPI communicator addresses as well). Therefore, a Fortran `integer` must be "large enough" to hold an address on the machine. For all 32-bit address space machines we have tested this is the case. On machines with a 64-bit address space, however, this may not be true. In particular, the size of a Fortran `integer` on the Silicon Graphics Power Challenge and DEC Alpha (but *not* the Cray T3D) is 32-bits and is not large enough to hold a machine address. The solution on these machines is to use the (nonstandard, but supported) Fortran declaration `integer*8` for the context variable.

Startup

The MPI standard provides for *source code* portability. However, the MPI standard does *not* specify how an MPI program shall be started or how the number of processes in the computation is specified. These will vary according to the computer being used and the choice of MPI implementation. The notes below are from our experiences testing PGAPack on different machines.

Silicon Graphics Challenge

The Silicon Graphics Challenge is a 32-bit symmetric multiprocessor. We used `MPICH` with the `ch_shmem` device and the `ncc` C compiler. Several warnings were received

```
warning(3262): parameter "ctx" declared and never referenced
warning(3141): cast between pointer-to-object and pointer-to-function
```

but the library was successfully built. To run a parallel PGAPack program, use either

```
a.out -np nprocs
```

or `MPICH`'s `mpirun` command.

Silicon Graphics Power Challenge

The Silicon Graphics Power Challenge is similar to the Challenge, except that it has a 64-bit address space. On this machine the size of an integer (`int` in C and `integer` in Fortran) is not the same as the size of an address. Fortran users should use the declaration `integer*8` for the context variable (and `integer` for other Fortran integer declarations). See also Chapter 11.

We used `MPICH` with the `ch_p4` device and the the MIPSpro C compiler (`cc`). We found a bug in `pca`, the Power C Analyzer, and recommend not using it for now. (To do this do not specify the `-pca` switch to `cc`). To run a parallel PGAPack program, use

```
a.out -np nprocs
```

or `MPICH`'s `mpirun` command.

Cray T3D

The Cray T3D has a 64-bit address space. However, the size of an integer on the T3D is the same as the size of an address, and therefore no special considerations are needed for declaring the context variable in Fortran.

On the T3D a C `double` is 64 bits. The Fortran `double precision` data type, however, is not supported (by default). One workaround is to declare all floating-point numbers `REAL`, as these are 64 bits on the T3D. The other workaround is to use the compiler switch "-dp".

To compile for a Cray T3D, cross compilation is done on a front-end machine (a Cray C90 in our case). Set Cray's `TARGET` environment variable so the compiler, linker, etc., will know which architecture to compile for.

```
setenv TARGET cray-t3d
```

An alternative is to use "-T cray-t3d" with `cc` and "-C cray-t3d" with `cf77`. Another alternative is to explicitly use the cross compilers (`/mpp/bin/cc` and `/mpp/bin/cf77`) and linker (`/mpp/bin/mppldr`).

We used the MPI in `/usr/local/mpp/lib/libmpi.a`. Adding `-lmpi` in your link step may also find the MPI library. If a successful T3D executable was built, the command "file a.out" should say "MPP absolute."

To run a parallel PGAPack program, use

```
a.out -npes nprocs
```

where `nprocs` is a power of two.

Intel Paragon

We used `MPICH` with the `ch_nx` device and compiled with `cc -nx`. To run a parallel PGAPack program, use

```
a.out -sz nprocs
```

or `MPICH`'s `mpirun` command.

IBM SP2

We tested the IBM SP2 using both `MPICH` with the `ch_eui` device, and IBM's research MPI, MPI-F. We compiled PGAPack with `xlc` and linked with `mpCC`. Execution required setting a number of environment variables. We were successful with the following, but this may vary with the system software installed on the SP you are using.

```
setenv MP_HOSTFILE /sphome/hostfile
setenv MP_PROCS       np
setenv MP_EUILIB      us
setenv MP_INFOLEVEL    0
setenv MP_HOLD_STDIN YES
setenv MP_PULSE        0
a.out
```

Convex Exemplar

We used `MPICH` with the `ch_shmem` device. Be sure to compile (the Fortran examples) with `fort77`, not `f77`. Also, you must link with `/usr/lib/libU77.a` *last* to satisfy `iargc` and `getarg`. This *must* be done *manually* in the prototype makefiles `./examples/fortran/Makefile.in` and `./examples/mgh/Makefile.in` *before* running `configure`. To run a parallel PGAPack program using `MPICH` use the `mpirun` command.

Sun SparcStation

We used `MPICH` with the `ch_p4` device and the GNU C compiler `gcc`. The `instverf` test program was run using 4 processes with:

```
/usr/local/mpi/bin/mpirun instverf -arch sun4 -np 4
```

Silicon Graphics Workstation

We used `MPICH` with the `ch_p4` device and `mpirun` command, the `cc` C compiler, and `f77` Fortran compiler.

IBM/RS6000 Workstation

We have successfully run PGAPack on both single workstations and networks of workstations using the `MPICH` implementation with the `ch_p4` device.

Hewlett Packard Workstation

We used `MPICH` with the `ch_shmem` device and `mpirun` command, the `gcc` C compiler, and `fort77` Fortran compiler.

DEC Alpha Workstation

DEC Alpha workstations have a 64-bit address space. On this machine the size of an integer (`int` in C and `integer` in Fortran) is not the same as the size of an address. Fortran users should use the declaration `integer*8` for the context variable (and `integer` for other Fortran integer declarations). See also Chapter 11.

Appendix E

Common Problems

- When reading input value to be used as parameters in `PGASet` calls, the `PGAset` calls themselves may not be executed until *after* `PGACreate` has been called.
- In C, when reading input parameters which are of type `double`, the `scanf` conversion specification should be of the form `%lf`, *not* `%f` which is appropriate for `float`s.
- An infinite loop can occur if the number of permutations of the bit string is less than the population size. For example, for a binary-valued string of length four, there are $2^4 = 16$ possibilities. If the population size is greater than 16, and duplicate strings are not allowed in the population, an infinite loop will occur.
- Erroneous results can occur if the name of a user's function conflicts with a library function used by PGAPack. For example, if a program defined its own `ceil` function, this would conflict with the C math library function of the same name.
- All floating point constants and variables used in PGAPack are of type `double`. Particularly from Fortran, the user should be careful to make sure that they pass a `double precision` constant or variable.
- `PGACreate` removes command line arguments. One consequence is that if `PGACreate` is called twice in the same program (unusual, but legal), the second `PGACreate` call will *not* receive the command-line arguments.
- If one includes `mpi.h` (or `mpif.h`) when it should not be, errors will result, as well as warnings about redefining macros and typedefs. This usually happens when a sequential version of PGAPack is used (with "fake" MPI stub routines and definitions) and the user's program explicitly includes "real" `mpi.h` or `mpif.h` header files.
- If one fails to include `mpi.h` (or `mpif.h`) when it should be (such as calling MPI functions directly) errors may result. Since `pgapack.h` includes `mpi.h` this should not happen in C. The Fortran include file, `pgapackf.h`, however, does *not* include `mpif.h`. The user must explicitly include it in every subroutine and function that makes MPI calls. Not including `mpif.h` could result in any of several different errors, including
  - syntax errors when compiling (for example, `MPI_COMM_WORLD` being undefined)
  - general errors in the computed results
  - the program crashing when it calls the undefined subroutine `MPI_Init`
  - general MPI errors such as:

    ```
    0 - Error in MPI_COMM_RANK : Invalid communicator
    [0] Aborting program!
    ```

We have also seen the following error from not including `bmpif.h` in the main program:

```
PGACreate: Invalid value of datatype: 0
PGAError: Fatal
```

- If the `ch_p4` device in `MPICH` is used to run on workstations one must have a correct processor group file (`procgroup`). The error message

  ```
  (ptera-36%)a.out
  p0_18429:  p4_error: open error on procgroup file (procgroup): 0
  (ptera-37%)
  ```

  may occur if the processor group file is not specified correctly. See the `MPICH` users guide for more details.

- A common error with the `procgroup` file when using the `ch_p4` device in `MPICH` is to have an incorrect path to the executable.

- When compiling the `examples` directory we have seen "multiply defined" error messages. For example:

  ```
  Making C examples
    Compiling classic
  ld: /usr/local/mpi/lib/sun4/ch_p4/libmpi.a(initialize.o): _MPI_Initialized: multiply defined
  collect2: ld returned 2 exit status
  ```

  We have seen this error occur when a sequential version of PGAPack was built and the library (`./lib/arch/libpgag.a` or `./lib/arch/libpga0.a`) was not deleted before attempting to build a new, parallel version of PGAPack. The "fake" MPI stub routines are in the sequential library and have name conflicts when a "real" MPI library is referenced. The solution is to delete the old `.a` file and rerun `make install`.

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